

$$\bar{V}_0 = (U \cos \theta - \dot{x} \sin \theta + \frac{1}{2} b \dot{\theta}) \hat{i} + (U \sin \theta + \dot{x} \cos \theta + (a+b) \dot{\theta}) \hat{j}$$

$$\alpha_0 \approx \frac{U \sin \theta + \dot{x} \cos \theta + (a+b) \dot{\theta}}{U \cos \theta - \dot{x} \sin \theta + \frac{1}{2} b \dot{\theta}} = \frac{U \theta + \dot{x} + (a+b) \dot{\theta}}{U - \cancel{\dot{x} \theta} + \cancel{\frac{1}{2} b \dot{\theta}}}$$

$$U \gg b \dot{\theta}$$

$$U \gg \dot{x} \theta$$

$$\alpha = \theta + \dot{x}/U + (a+b) \dot{\theta}/U$$

Kinetic

$$T^* = \frac{1}{2} m v_c^2 + \frac{1}{2} I_c \dot{\theta}^2 = \frac{1}{2} m (U^2 + \dot{x}^2 + a^2 \dot{\theta}^2 + 2aU\dot{\theta}\theta + 2a\dot{x}\dot{\theta}) + \frac{1}{2} I_c \dot{\theta}^2$$

Potential Energy

$$V = \frac{1}{2} k x^2$$

For a 2 DoF system, there are 2 Lagrange Equations:

$$\frac{d}{dt} \frac{\partial T^*}{\partial \dot{\theta}} - \frac{\partial T^*}{\partial \theta} + \frac{\partial V}{\partial \theta} = \underline{\underline{\epsilon}}_{\theta}, \quad \frac{d}{dt} \frac{\partial T^*}{\partial \dot{x}} - \frac{\partial T^*}{\partial x} + \frac{\partial V}{\partial x} = \underline{\underline{\epsilon}}_x$$

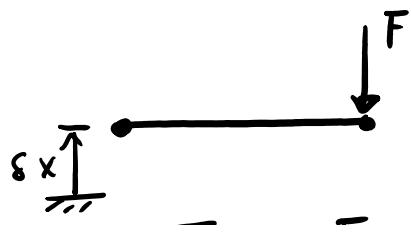
Generalized forces

$$\delta W = \underline{\underline{\epsilon}}_{\theta} \delta \theta$$



$$= -F(a+b)$$

$$\delta W = \underline{\underline{\epsilon}}_x \delta x = -F \delta x$$



$$\underline{\underline{\Sigma}}_{\theta} = -F(a+b)$$

$$\begin{array}{c} \uparrow \\ \underline{\underline{\Sigma}}_x = -F \end{array}$$

$$\frac{\partial T^*}{\partial \dot{\theta}} = m(a^2\dot{\theta} + aU\dot{\theta} + a\dot{x}) + I_c\ddot{\theta} \stackrel{d/dt}{=} ma^2\ddot{\theta} + maU\dot{\theta} + m\ddot{a}\dot{x} + I_c\ddot{\theta}$$

$$\frac{\partial T^*}{\partial \theta} = maU\dot{\theta}$$

$$\underline{\underline{\Sigma}}_{\theta} = -C_L\left(\theta + \frac{\dot{x}}{a} + \frac{(a+b)}{a}\dot{\theta}\right)(a+b)$$

$$\frac{\partial V}{\partial \theta} = 0 \quad (I_c + ma^2)\ddot{\theta} + m\ddot{a}\dot{x} + \cancel{maU\dot{\theta}} - \cancel{maU\dot{\theta}} + 0 =$$

$$T^* = \frac{1}{2}mv_c^2 + \frac{1}{2}I_c\dot{\theta}^2 = \frac{1}{2}m(U^2 + \dot{x}^2 + a^2\dot{\theta}^2 + 2aU\dot{\theta}\theta + 2a\dot{x}\dot{\theta}) + \frac{1}{2}I_c\dot{\theta}^2$$

$$V = \frac{1}{2}Kx^2$$

$$\frac{\partial T^*}{\partial \dot{x}} = m(\dot{x} + a\dot{\theta}) \stackrel{d/dt}{=} m\ddot{x} + ma\ddot{\theta}$$

$$\frac{\partial T^*}{\partial x} = 0$$

$$\frac{\partial V}{\partial x} = Kx$$

$$\underline{\underline{\Sigma}}_x = -C_L\left(\theta + \frac{\dot{x}}{a} + \frac{(a+b)}{a}\dot{\theta}\right)$$

$$m\ddot{x} + ma\ddot{\theta} - 0 + Kx = -C_L\left(\theta + \frac{\dot{x}}{a} + \frac{(a+b)}{a}\dot{\theta}\right)$$

$$(I_c + ma^2)\ddot{\theta} + m\ddot{a}\dot{x} = -C_L\left(\theta + \frac{\dot{x}}{a} + \frac{(a+b)}{a}\dot{\theta}\right)(a+b)$$

$$(I_c + ma^2)\ddot{\theta} + m a \ddot{x} = -C_2 \left(\theta + \frac{\dot{x}}{u} + \frac{(a+b)}{u} \dot{\theta} \right) (a+b)$$

Put in canonical form: $M \ddot{y} + C \dot{y} + K y = 0$ $\bar{y} = \begin{bmatrix} \theta \\ x \end{bmatrix}$

M

C

K

$$\begin{bmatrix} I_c + ma^2 & ma \\ ma & m \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} C_2 \frac{(a+b)^2}{u} & C_2 \frac{a+b}{u} \\ C_2 \frac{a+b}{u} & C_2 / u \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} C_2(a+b) & 0 \\ C_2 & K \end{bmatrix} \begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$