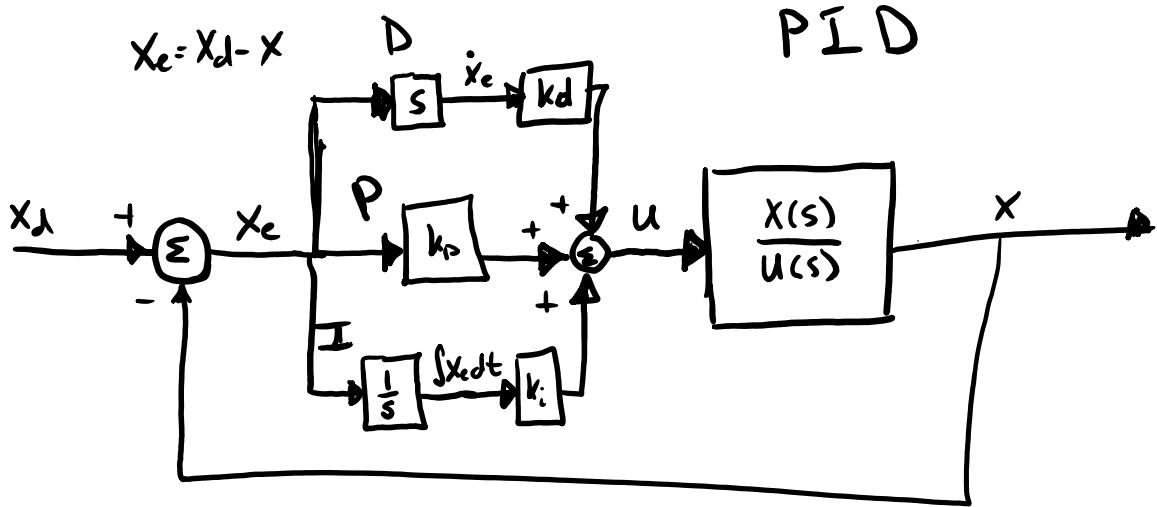


# PID Control

$$sX(s) \Rightarrow \dot{x}(t)$$



$\frac{X(s)}{X_d(s)}$  closed loop

$$m\ddot{x} + c\dot{x} + kx = u$$

$$m\ddot{x} + c\dot{x} + kx = k_p(x_d - x) + k_i \int_0^t (x_d(\tau) - x(\tau)) d\tau + k_d (\dot{x}_d - \dot{x})$$

Laplace Transform

$$ms^2 X(s) + cs X(s) + k X(s) = k_p (X_d(s) - X(s)) + \frac{k_i}{s} (X_d(s) - X(s)) + k_d s [X_d(s) - X(s)]$$

$$ms^2 X(s) + \underbrace{[c + k_d] s X(s)}_{\text{damping}} + \underbrace{[k + k_p]}_{\text{stiffness}} X(s) + \underbrace{\frac{k_i}{s} X(s)}_{\text{Steady error}} =$$

damping

stiffness

steady error

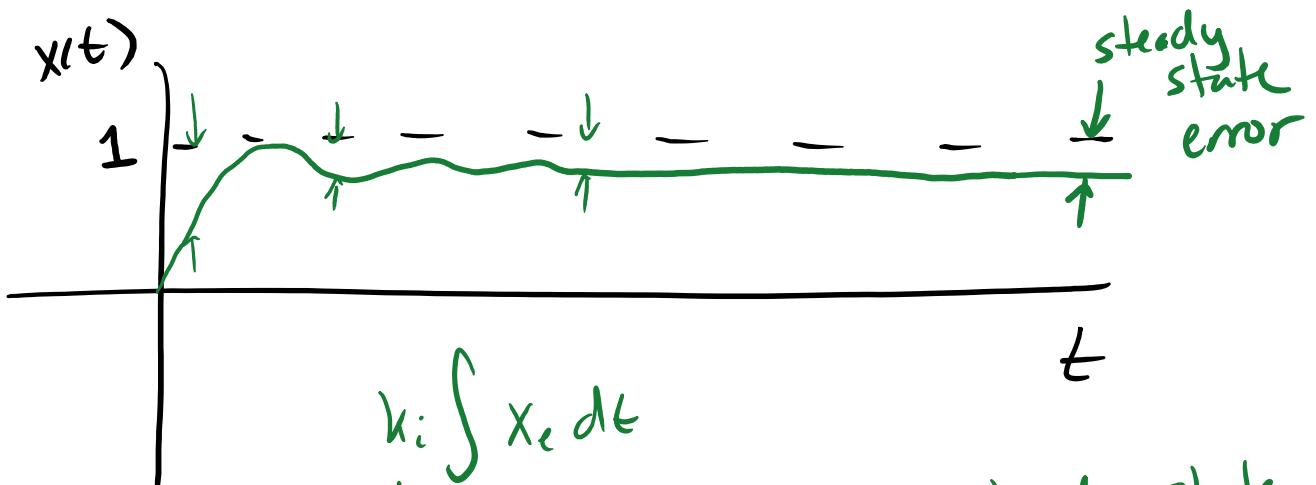
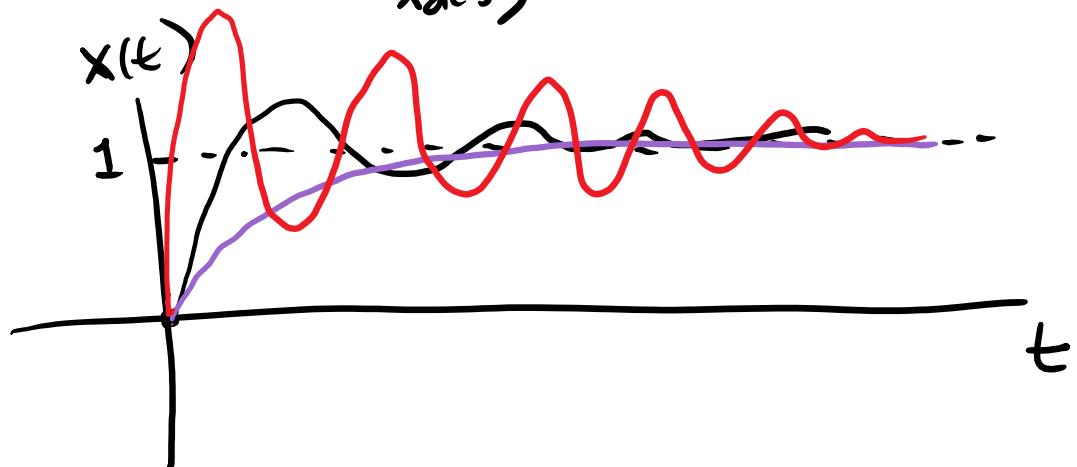
$$k_p X_d(s) + \frac{k_i}{s} X_d(s) + k_d s X_d(s)$$

$$\frac{X(s)}{X_d(s)} = \frac{k_p + \frac{k_i}{s} + k_d s}{ms^2 + (c+k_d)s + [k+k_p] + \frac{k_i}{s}}$$

closed loop  
TF

let you affect stability  
and characteristic dynamics

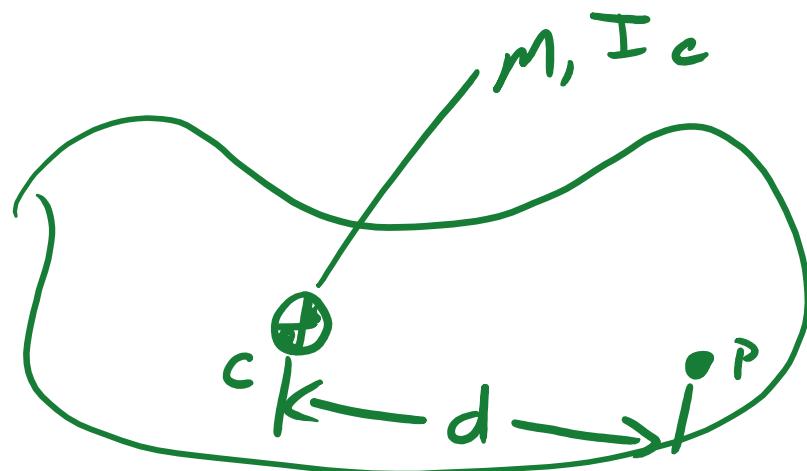
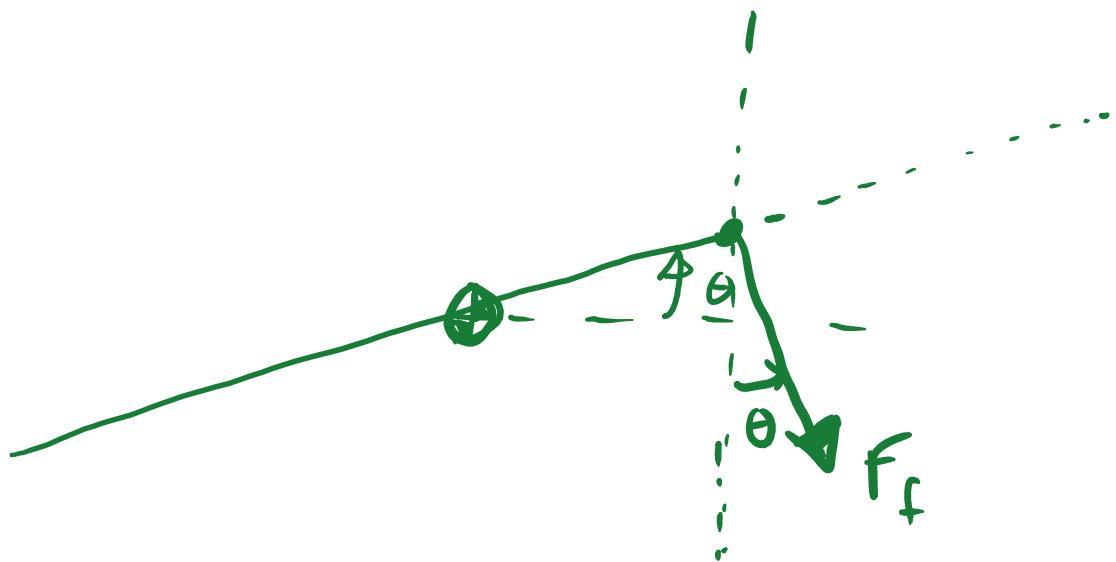
Step response of  $\frac{X(s)}{X_d(s)}$



$$k_i \int x_e dt$$

↳ used to reduce steady state error

error

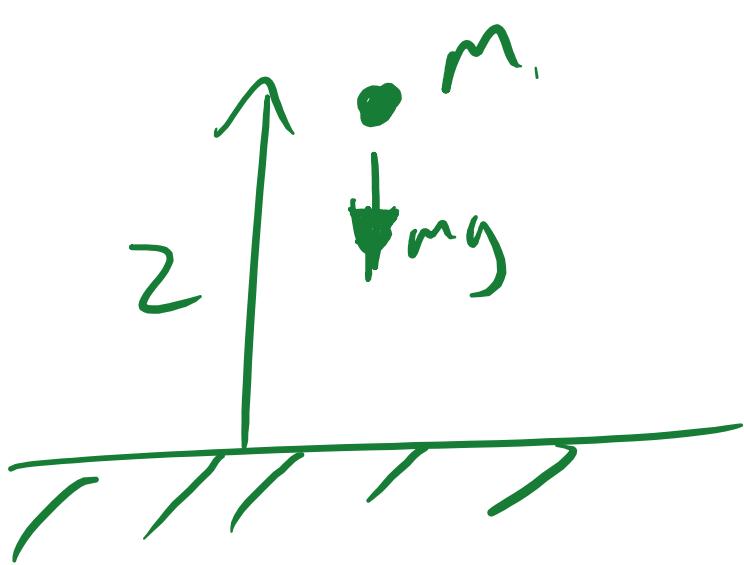


$$I_p = I_c + m d^2$$



$$I_c \ddot{\varphi} = \sum M_c$$

$$I_p \ddot{\theta} = \sum M_p$$



$$m\ddot{z} = -mg$$

$$\ddot{z} = -g$$

$V = \dot{z}$

$\dot{V} = -g$