

Chapter 3: Stability

Q: In terms of dynamics system, what is stability?

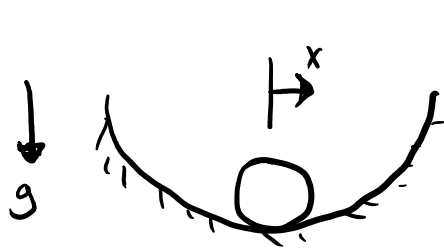
- tendency of disturbed system to return to a steady state (equilibrium)
- static + dynamic stability
- differential equation: sign of terms
- Routh criteria: method for investigating eigenvalues

Equilibrium: state of no acceleration

↳ configuration and motion of the system

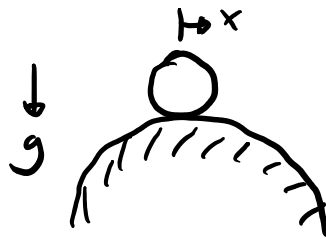
"Basic motion"

Static stability: will small deviation from equilibrium tend to cause movement back towards the equilibrium



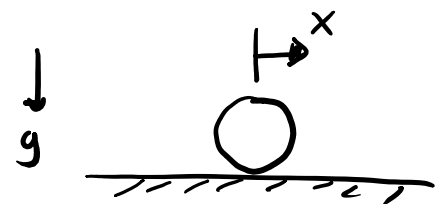
statically
stable

$x=0$ equilibrium



statically
unstable

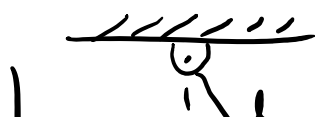
$x=0$ equilibrium



neutrally statically
stable

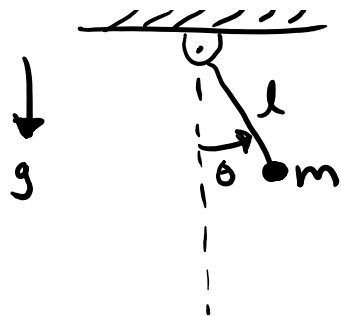
all x equilibrium

Simple pendulum



$\theta=0, \dot{\theta}=\ddot{\theta}=0$ equilibrium point

$$ml^2 \ddot{\theta} = -mgl \sin \theta$$



$$ml^2 \ddot{\theta} = -mgl \sin \theta$$

assume θ is small

$$ml^2 \ddot{\theta} + mgl \theta = 0$$

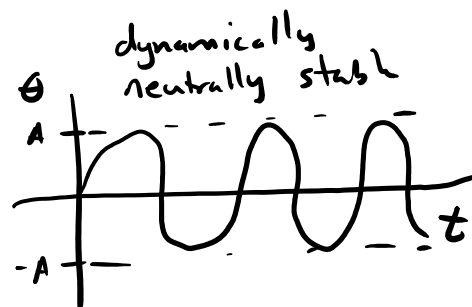
$$\ddot{\theta} + \frac{g}{l} \theta = 0$$

Three possible solutions:

complex number case

$$\frac{g}{l} > 0 \Rightarrow \theta(t) = A \sin\left(\sqrt{\frac{g}{l}} t + \phi\right)$$

$\omega_n = \sqrt{\frac{g}{l}}$ natural frequency

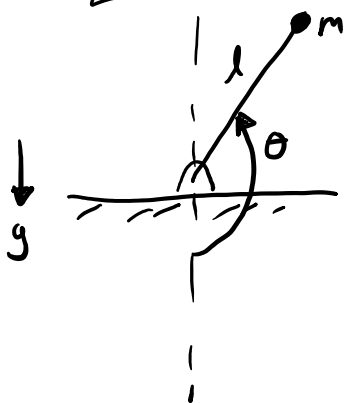


$$\frac{g}{l} = 0 \Rightarrow \theta(t) = 0$$



$$\frac{g}{l} < 0 \Rightarrow \theta(t) = A e^{\sqrt{\frac{g}{l}} t}$$

inverted pendulum



statically unstable



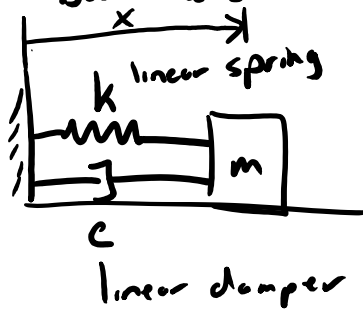
$\theta = 0, \pi, 2\pi, 3\pi, \dots$ } all equilibrium points
 $\dot{\theta} = \ddot{\theta} = 0$

$$\ddot{\theta} + \frac{g}{l} \theta = 0$$

static stability

coefficient of the coordinate directly determines static stability

What if the system has both restoring force/torque
but also a dissipative force/torque?

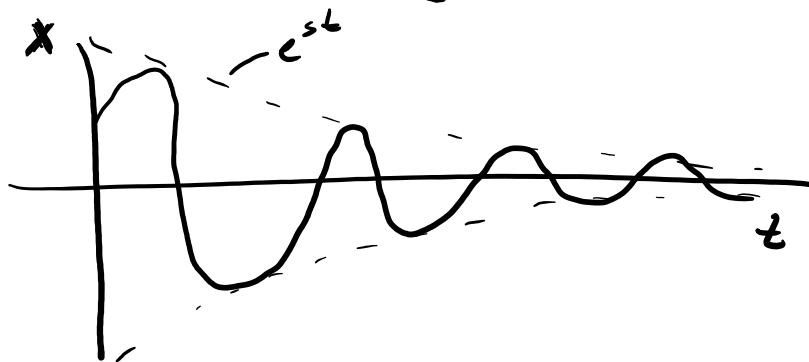


$$m\ddot{x} + c\dot{x} + kx = 0$$

If $m > 0$ and $k > 0$, then we can investigate
 c 's effect on the motion.

$c = 0 \Rightarrow$ statically stable; dynamically: neutrally stable

$c > 0 \Rightarrow$ statically stable, dynamically: stable



$$x(t) = Ae^{st} \sin(\omega t + \phi)$$

$c < 0 \Rightarrow$ static stable, dynamically instability

