

Canonical Form:  $M\ddot{x} + C\dot{x} + Kx = 0$

$$\bar{x} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$$

$$\begin{bmatrix} m & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} \frac{c_f + c_r}{u} & \frac{c_f a - c_r b}{u} \\ \frac{a c_f - b c_r}{u} & \frac{a^2 c_f + b^2 c_r}{u} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 & -(c_r + c_f) \\ 0 & b c_r - a c_f \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} = 0$$

Char. Eq:  $\det |Ms^2 + Cs + K| = 0$

↓ algebra

$$mI_2 s^4 + [m(a^2 c_f + b^2 c_r)/u + I_2(c_f + c_r)/u] s^3 +$$

$$[(c_f + c_r)(a^2 c_f + b^2 c_r)/u^2 - (a c_f - b c_r)^2/u^2 - m(a c_f - b c_r)] s^2 +$$

$$\underbrace{0s^4 + 0s^3}_{} = 0$$

associated with zero eigenvalues

Divide by  $s^2$ :

$$\checkmark \quad \begin{array}{c} a, b, u > 0 \\ \checkmark \end{array} \quad mI_2 s^2 + [m(a^2 c_f + b^2 c_r)/u + I_2(c_f + c_r)/u] s +$$

$$[(c_f + c_r)(a^2 c_f + b^2 c_r)/u^2 - (a c_f - b c_r)^2/u^2 - m(a c_f - b c_r)] = 0$$

zero eigenvalue

$Ae^{st} = Ae^{0t} = A \Rightarrow$  constant motion w.r.t time

$Ae^{st} = Ae^{ot} = A \Rightarrow$  constant motion wrt time

$y, \psi \Rightarrow$  ignorable coordinates, they don't affect the dynamics

For a quadratic polynomial, if all coefficients are positive the system is stable. ( $1^{\text{st}}$  step of R-H)

The  $s^0$  term reduces to:

$$\frac{(a+b)^2 C_f C_r}{U^2} + m(bC_r - aC_f) > 0 \text{ for stability}$$

$$a, b, C_f, C_r, m, U > 0$$

If  $m(bC_r - aC_f) > 0 \Rightarrow$  always stable



$bC_r > aC_f \Rightarrow$  always stable @ all speeds

As  $a$   $U$  increases the first term goes to zero.

So @ high speeds if  $bC_r < aC_f$  system will be unstable.

$bC_r > aC_f$  is called an understeer automobile

rear end has more lat. force generation ability than the front end

... called an oversteer automobile

$bC_r < aC_f$  is called an oversteer automobile  
front end has more lat. force gen.  
ability than the rear

$bC_r = aC_f$  is called a neutral steer automobile

If the automobile is oversteer ( $bC_r < aC_f$ )  
then there must be a speed when 

$$\frac{(a+b)^2 C_f C_r}{U^2} < m \underbrace{(aC_f - bC_r)}_{\text{positive}}$$

causes instability.

Solve for  $U$

$$U_{\text{crit}}^2 = \frac{(a+b)^2 C_f C_r}{m(aC_f - bC_r)} \quad \text{critical speed}$$

If oversteer and  $U > U_{\text{crit}} \Rightarrow \text{unstable} !!$

### Conclusions

- over-, under-, neutral steer only depend on  $a, b, C_r, C_f$
- $\psi, y$  ignorable coordinates
- if oversteer, system is unstable above  $U_{\text{crit}}$

Same model using body fixed coordinate and with steering

"Bicycle model of the car"

