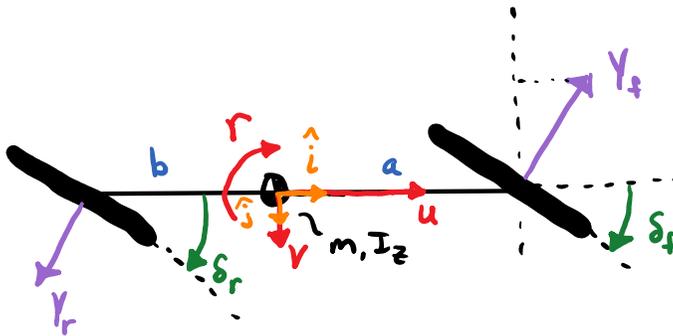


# Same model using body fixed coordinates and including steering

Bicycle model of the car



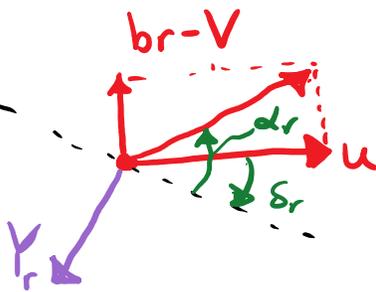
- $U$ : longitudinal velocity
- $V$ : lateral velocity
- $r$ : yaw angular rate
- $\delta_f, \delta_r$ : front and rear steer angles
- $Y_f, Y_r$ : front and rear lateral tire forces

## Chapter 2 Body Fixed EoMs

$$m(\dot{V} + rU) = Y_r \cos \delta_r - Y_f \cos \delta_f \approx Y_r - Y_f$$

$$I_z \dot{r} = -a Y_f \cos \delta_f - b Y_r \cos \delta_r \approx -a Y_f - b Y_r$$

### Slip angle of the rear



$$Y_r = C_{\alpha} \alpha_r$$

$$\alpha_r = \frac{br - V}{u} + \delta_r$$

front: 
$$\alpha_f = \frac{V + ar}{u} - \delta_f$$

$$(1) m(\dot{V} + rU) = -(c_f + c_r) \frac{V}{u} + (c_r b - c_f a) \frac{r}{u} + c_f \delta_f + c_r \delta_r$$

$$(2) I_z \dot{r} = (-a c_f + b c_r) \frac{V}{u} - (a^2 c_f + b^2 c_r) \frac{r}{u} + a c_f \delta_f - b c_r \delta_r$$



The Laplace Transform converts a linear ODE to algebraic equations that are functions of  $s$ .  
(equivalent to assuming solution  $x(t) = Ae^{st}$ )

2<sup>nd</sup> order linear ODE for single input, single output (SISO)

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t)$$

↓ Laplace Transform

$$ms^2 X(s) + cs X(s) + k X(s) = F(s)$$

$$X(s) \underbrace{[ms^2 + cs + k]}_{\text{char. eq}} = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \quad \left. \vphantom{\frac{X(s)}{F(s)}} \right\} \text{transfer function that relates } x(t) \text{ and } f(t)$$

TF =  $\frac{\text{numerator}}{\text{denominator}}$  → tied to the forcing term, describes the forced dynamics  
→ always the char. eq. that describes the unforced dynamics

roots of the denominator all called the "poles"

roots of the numerator all called the "zero"