

It is common to evaluate the yaw rate behavior and the lateral acceleration behavior for performance and rider comfort. $\dot{V} \Rightarrow sV(s)$ and remember $m(\dot{V} + ru)$

$$\frac{sV(s) + r(s)U}{\delta_f(s)} = \frac{I_z C_f s^2 + ((a+b)bC_f C_r/U)s + (a+b)C_f C_r}{\Delta}$$

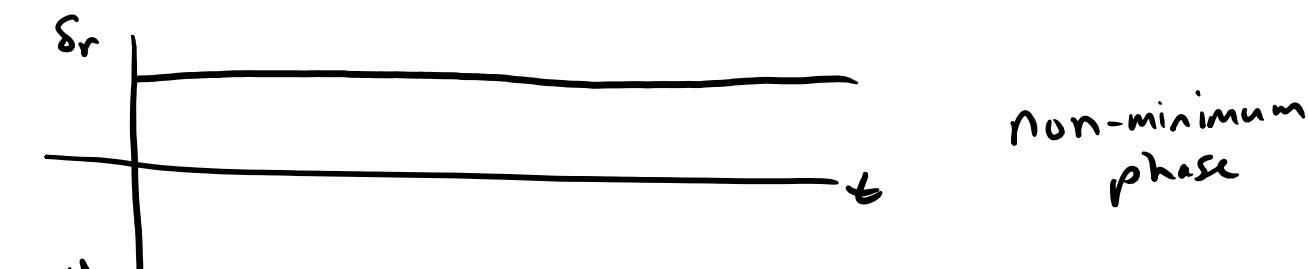
$$\frac{sV(s) + r(s)U}{\delta_r(s)} = \frac{I_z C_r s^2 + [(a+b)a C_f C_r/U]s - (a+b)C_f C_r}{\Delta}$$

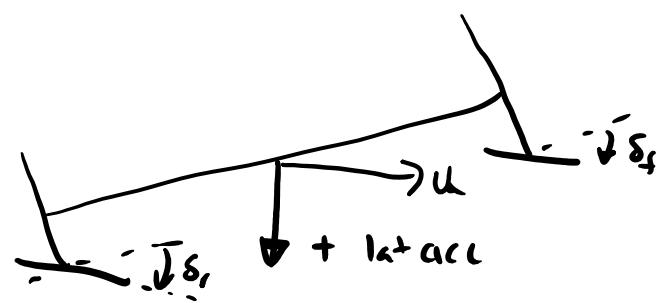
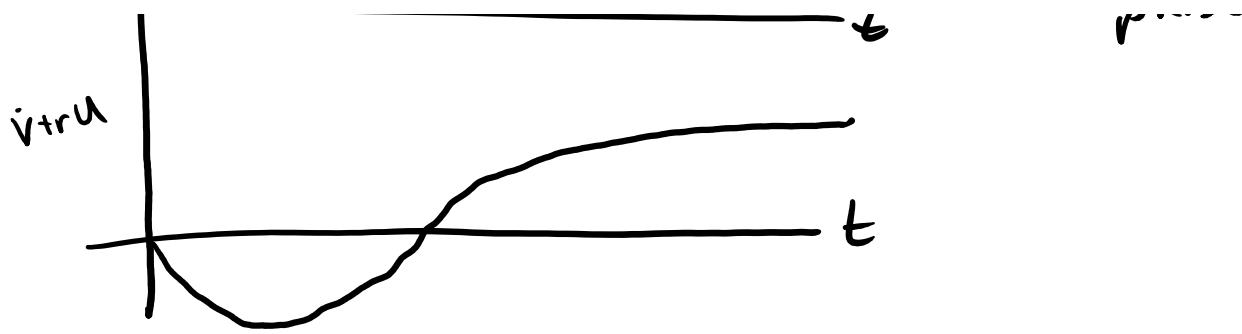
$$G_{af} \triangleq \left. \frac{sV(s) + r(s)U}{\delta_f(s)} \right|_{s=0} = \frac{U^2}{(a+b) + Ku^2}$$

$$G_{ar} \triangleq \frac{-U^2}{(a+b) + Ku^2}$$

Notes:

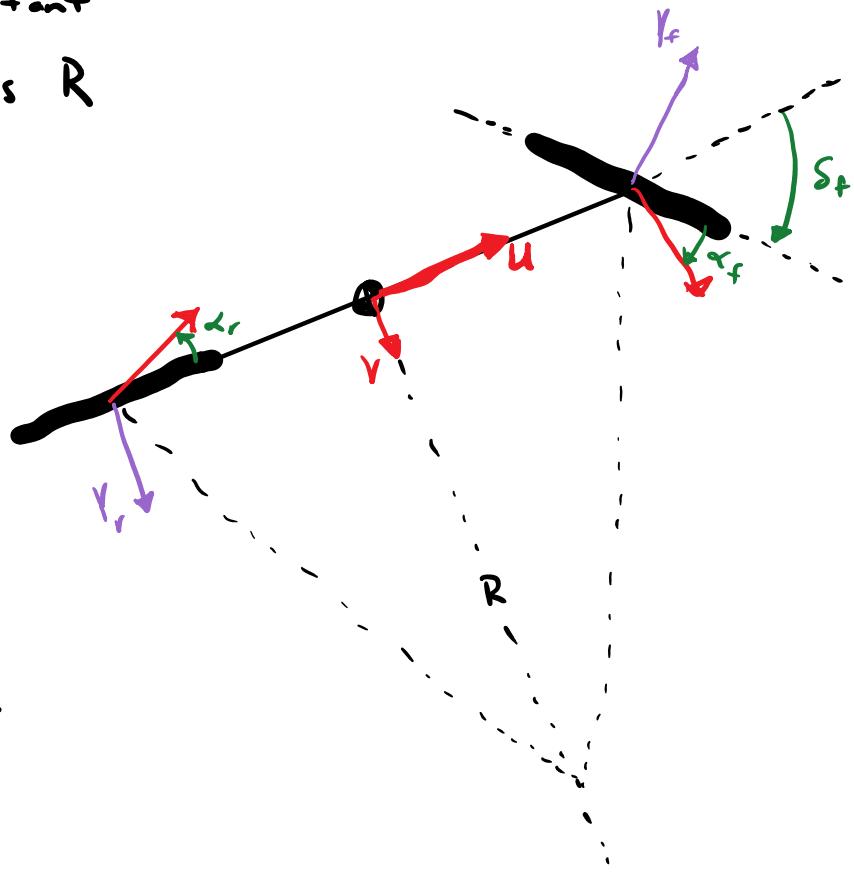
- num. and den. have the same order, so no attenuation at high frequencies
- for the δ_r lat. acc. TF there is a single right-half plane zero. This indicates we have a "non-minimum phase" system. Steering to the right (positive) will initially cause a lateral acceleration to the left.





Steady Turns

- U, V, r are all constant
- constant turn radius R
- tangential acc. = 0
- centripetal acc. = $\frac{U^2}{R}$



Recall slip angles

$$\alpha_f = \frac{V + ar}{U} - \delta_f$$

$$\alpha_r = \frac{br - V}{U}$$

↓ eliminate V

$$\delta_f = \frac{(a+b)r}{U} - (\alpha_f + \alpha_r)$$

$$\text{If } R \gg (a+b) \quad \text{then} \quad r \approx \frac{U}{R}$$

$$\delta_f = \frac{(a+b)}{R} - (\alpha_f + \alpha_r) \quad (3)$$

EoM

$$m(\vec{V} + r\vec{U}) = \vec{V}_r - \vec{V}_f \Rightarrow m \frac{\vec{U}}{R} = \vec{V}_r - \vec{V}_f \quad (1)$$

centripetal acc.

$$m(\ddot{X} + r\dot{U}) = Y_r - Y_f \Rightarrow m \frac{\ddot{X}}{R} = r_r - r_f \quad \textcircled{1}$$

$$I_2 \ddot{x}^o = -Y_f a - Y_r b \Rightarrow Y_f a = -Y_r b \quad \textcircled{2}$$

Solve $\textcircled{1}$ + $\textcircled{2}$ for $Y_r + Y_f$.

$$Y_f = \frac{-mU^2 b}{R(a+b)} \quad Y_r = \frac{mU^2 a}{R(a+b)}$$

$$\text{If } Y_f = C_f \alpha_f \quad Y_r = C_r \alpha_r$$

Substitute for $\alpha_f + \alpha_r$ in $\textcircled{3}$

$$\delta_f = \frac{a+b}{R} - \left(\frac{Y_f}{C_f} + \frac{Y_r}{C_r} \right) = \frac{a+b}{R} + \frac{U^2}{R} \frac{m}{(a+b)} \left[\frac{bC_r - aC_f}{C_f C_r} \right]$$

$$\delta_f = \frac{a+b}{R} + K \frac{U^2}{R} \quad !$$

If $K=0 \Rightarrow$ neutral steer

$$\delta_f = \frac{a+b}{R}$$

If $K > 0 \Rightarrow$ under steer

δ_f increases linearly with $\frac{U^2}{R}$

If $K < 0$

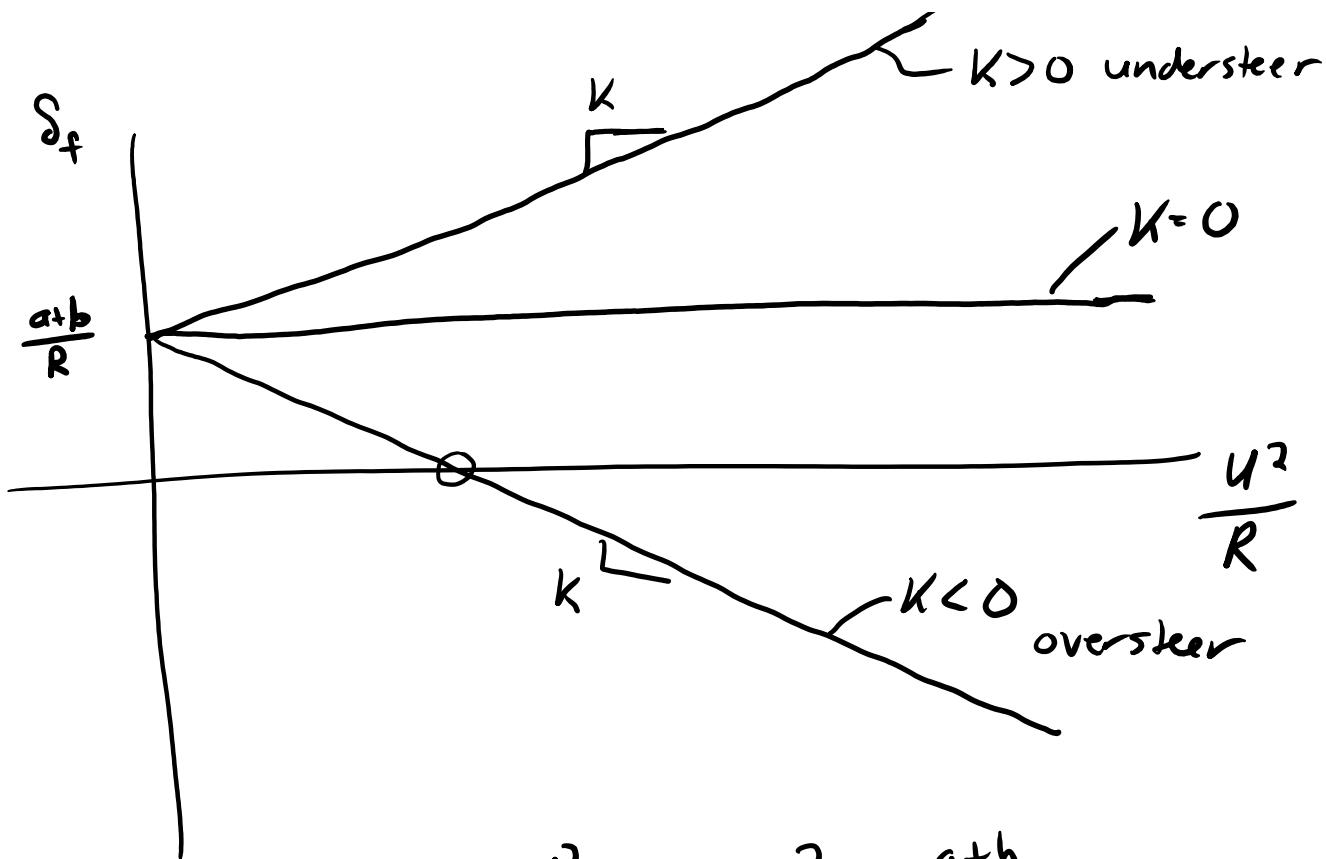
δ_f decreases linearly with $\frac{U^2}{R}$

If $|K \frac{U^2}{R}| > \frac{a+b}{R}$ steer angle is negative!

?

K

 $K > 0$ understeer



$$\delta_f = 0 = \frac{a+b}{R} + K \frac{U^2}{R} \Rightarrow U^2 = -\frac{a+b}{K}$$

$$U^2 = -\frac{(a+b)^2 C_f C_r}{m(bC_r - aC_r)} \quad \text{some } U_{crit} \text{ from the stability analysis}$$

Above the critical speed 1) system is unstable
 & 2) have to steer in opposite direction.

"counter steering"