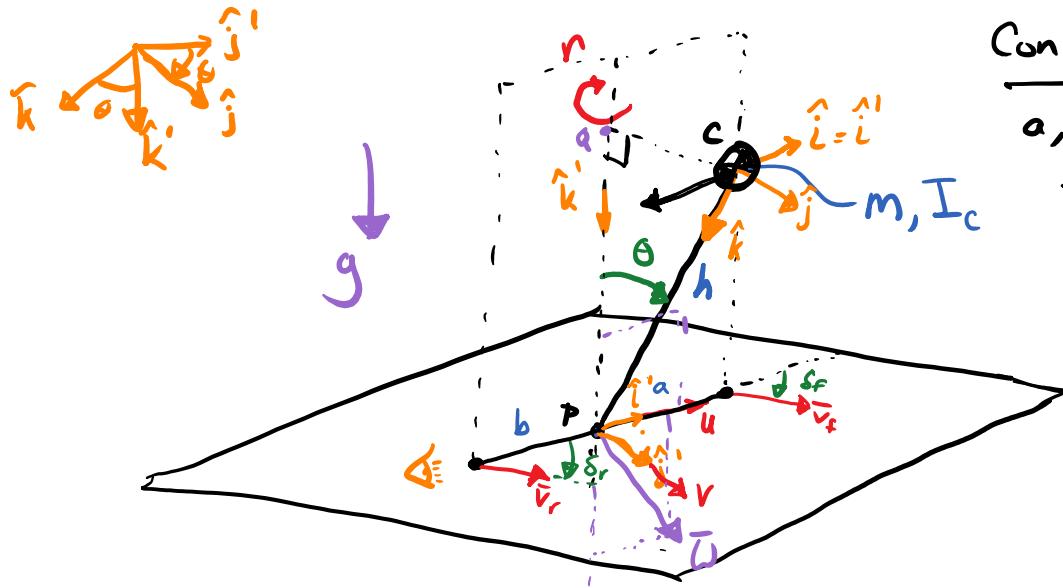


Dynamic Equations of Motion



Time Varying Variables

$$\theta, \dot{\theta}, V, r, S_f, S_r$$

Constant Parameters

$$a, b, U, R, m, I_c$$

$$I_c = \begin{bmatrix} I_{ii} & 0 & 0 \\ 0 & I_{jj} & 0 \\ 0 & 0 & I_{kk} \end{bmatrix}$$

Basic Motion

$$\theta, \dot{\theta}, V, r = 0$$

Kinetic Energy

$$T = \frac{1}{2} m |\vec{v}_c|^2 + \frac{1}{2} (I_i \omega_i^2 + I_j \omega_j^2 + I_k \omega_k^2)$$

$$\bar{\omega} = \begin{bmatrix} \omega_i \\ \omega_j \\ \omega_k \end{bmatrix}$$

Works if you have principal MoI
3D angular velocity vector of the RB

Potential energy

$$V = mgh\cos\theta$$

Generalized Forces

$$\sum F_g = 0$$

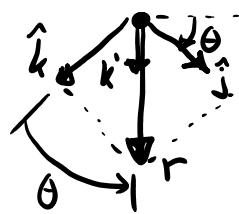
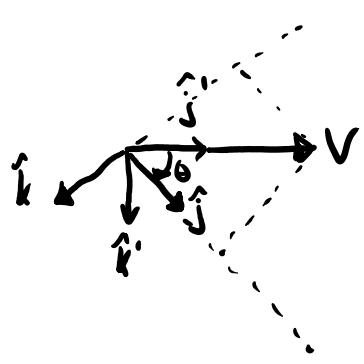
$$\vec{v}_c = \vec{v}_p + \bar{\omega} \times \vec{r}^{P-C}$$

Write the velocities in the BF coordinate system $\hat{i}, \hat{j}, \hat{k}$

$$\hat{i}^{\parallel}$$

$$\bar{V}_c = \begin{bmatrix} U \\ V\cos\theta \\ -V\sin\theta \end{bmatrix} + \begin{bmatrix} \dot{\theta} \\ r\sin\theta \\ r\cos\theta \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -h \end{bmatrix}$$

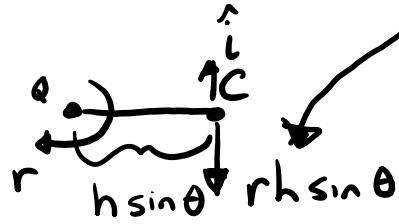
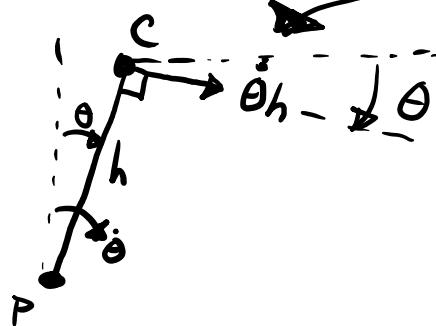
\hat{i}	\hat{j}	\hat{k}
$\dot{\theta}$	$r\sin\theta$	$r\cos\theta$
0	0	-h



$$\bar{\omega} \times \bar{r}^{PC} = -rh\sin\theta \hat{i}$$

$$+ \dot{\theta}h\hat{j}$$

$$+ 0\hat{k}$$



$$\bar{V}_c = \begin{bmatrix} U - rh\sin\theta \\ V\cos\theta + \dot{\theta}h \\ -V\sin\theta \end{bmatrix}$$

$$T = \frac{1}{2}m \left[(U - rh\sin\theta)^2 + (V\cos\theta + \dot{\theta}h)^2 + (-V\sin\theta)^2 \right]$$

$$+ \frac{1}{2} \left[I_i \dot{\theta}^2 + I_j (r\sin\theta)^2 + I_k (r\cos\theta)^2 \right]$$

Note that V and r are determined by a, b, U, R, S_f, S_r , so θ is the only coordinate and we 1 DoF system. Remember $\Sigma \ddot{\theta} = 0$ (no slip).

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\theta}} - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = 0$$

$$mh\cos\theta\dot{v} - mhV\sin\theta\dot{\theta} + mh^2\ddot{\theta} + I_i\ddot{\theta} + mhr\cos\theta(u - hr\sin\theta) - mVs\in\theta(V\cos\theta + h\dot{\theta}) + mV^2\sin\theta\cos\theta + I_jr^2s\in\theta\cos\theta - I_kr^2s\in\theta\cos\theta - mgh\sin\theta = 0$$

Linearize: $\sin\theta \approx \theta, \cos\theta \approx 1, rV \approx 0, V\dot{\theta} \approx 0, r^2 \approx 0$, etc.

Linear EoM

$$(I_i + mh^2)\ddot{\theta} - mgh\theta = -mh(\dot{v} + ru)$$