

## Longitudinal Dynamics

Typical aircraft have two primary modes of motion associated with the longitudinal dynamics (tied to two pairs of complex eigenvalues).

Long period mode (Phugoid mode): very slow oscillation in altitude and speed and has little to no damping (could be unstable). Period is from 20s to 60s. Little change in angle of attack.

Short period mode: highly damped mode in pitch, period is 0.5s to 5s.

Modes are found by writing the linear EOM for altitude, pitch, and forward velocity, and then solving for the eigenvalues and eigenvectors.

The Phugoid mode can be estimated with a simpler analysis that focuses on energy transfer from speed variations to altitude variations.

Assumptions:

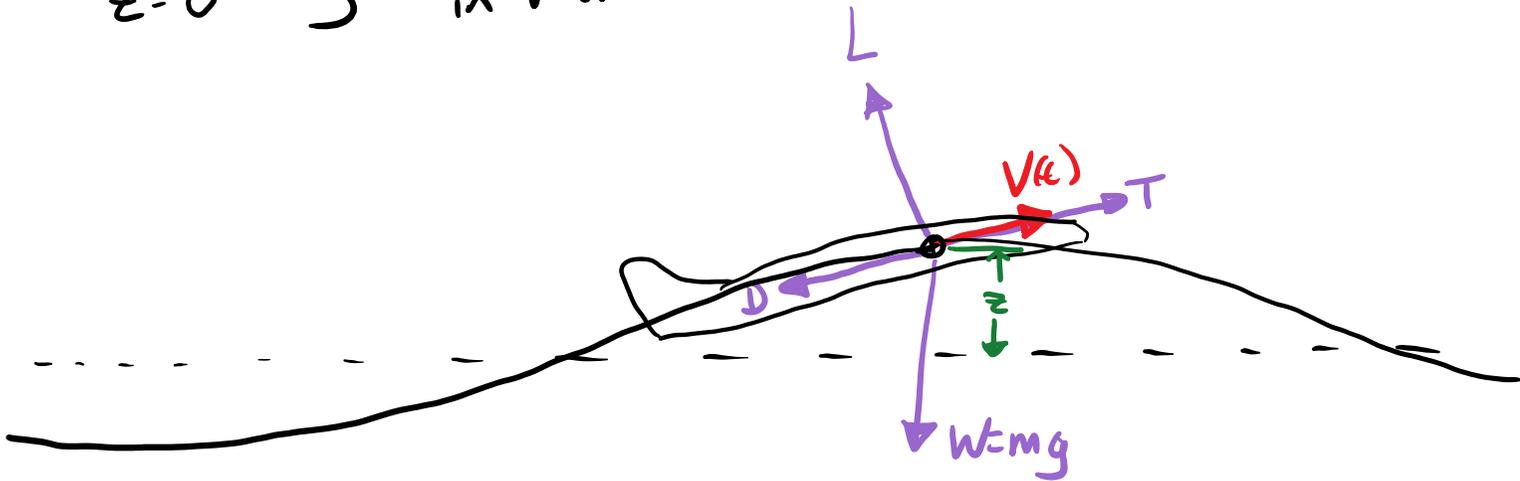
- small angle of attack (decoupling altitude & velocity from pitch)
- energy is conserved

Basic motion:

$$\text{pitch} = 0$$

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$$\left. \begin{aligned} V(t) &= V_0 \\ z &= 0 \end{aligned} \right\} \text{small deviations} \\ \text{in } V \text{ and } z$$



## Conservation of Total Energy

$$\underbrace{\frac{1}{2} m V_0^2}_{\text{kinetic energy @ basic motion}} = \underbrace{\frac{1}{2} m V^2}_{\text{kinetic @ perturbed state}} + \underbrace{mgz}_{\text{potential at perturbed state}}$$

rearrange to  $V^2 = V_0^2 - 2gz$  (1)

Lift @ basic motion

$$L = mg$$

If the lift is increased or decreased due to velocity changes.

$$L = \frac{\rho V^2}{2} C_L S$$

$$1 \dots \left( \frac{V}{V_0} \right)^2 \quad (2)$$

$$L = mg \left( \frac{v}{v_0} \right)^2 \quad (2)$$

Combine (1) + (2)

$$L = mg \left( 1 - \frac{2gz}{v_0^2} \right)$$

Newton's Law for the vertical motion:

$$m \ddot{z} = L - mg = mg \left( 1 - \frac{2gz}{v_0^2} \right) - mg$$

$$\ddot{z} + \frac{2g^2}{v_0^2} z = 0 \quad \text{undamped oscillator}$$

$$\omega_n = \sqrt{2} \frac{g}{v_0} \quad \text{natural frequency}$$

$$T = \frac{2\pi}{\omega_n} = \sqrt{2} \frac{\pi v_0}{g}$$

That's All Folks!