

Parameters (constant)
 a, b, l, u, I_z

Variables

$\ell(t)$
 $y(t), \dot{y}(t)$
 $F(t)$
 $x(t)$

Nominal (Basic) Motion

$$\dot{x} = u, \dot{y} = 0, \ell = \dot{\ell} = 0, F = 0$$

Perturbed Motion

$$\ell \text{ is small} \Rightarrow \cos \ell \approx 1, \sin \ell \approx \ell$$

$$\dot{y} \text{ is small} \quad \dot{y}/u \ll 1$$

EOMs

$$\left. \begin{aligned} \sum F_x &= m a_x \\ \sum F_y &= m a_y \\ \sum M_z &= I_z \alpha_z \end{aligned} \right\}$$

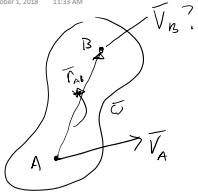
$$\ddot{x} = 0 \text{ because } \dot{x} = u$$

$$\left. \begin{aligned} F &= F \cos \ell = m \ddot{y} \\ -F b &= I_z \ddot{\ell} \end{aligned} \right\} \text{EOM}$$

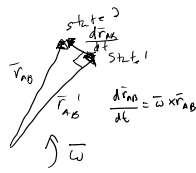
$$m \ddot{y} = F$$

$$I_z \ddot{\ell} = -F b$$

What is F?



$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{AB}$$



$$1) m \ddot{y} = F$$

$$2) I_2 \ddot{\varphi} = -Fb$$

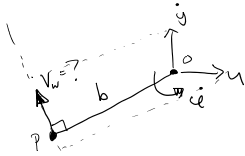
$$3) \dot{y} u \ell - b \dot{\varphi} = 0$$

diff 3)

$$\ddot{y} - u \dot{\varphi} - b \ddot{\varphi} = 0$$

$$\ddot{y} = u \dot{\varphi} + b \ddot{\varphi}$$

$$m(u \dot{\varphi} + b \ddot{\varphi}) = F = \frac{-I_2 \ddot{\varphi}}{b}$$



lateral velocity at wheel contact

$$v_w = \dot{y} \cos \ell - u \sin \ell - b \dot{\varphi} \quad \text{non-linear}$$

$$v_w \approx \dot{y} - u \ell - b \dot{\varphi} \quad \text{linear}$$

no-slip! $v_w = 0 = \dot{y} - u \ell - b \dot{\varphi}$

kinematic constraint

$$(I_2 + mb^2) \ddot{\varphi} + mb u \dot{\varphi} = 0$$

$r = \dot{\varphi}$ yaw rate

$$(I_2 + mb^2) \dot{r} + mb u r = 0$$

$$\dot{r} + \frac{mbu}{I_2 + mb^2} r = 0$$

$$\dot{r} + A r = 0$$

$$A = \frac{mbu}{I_2 + mb^2}$$

linear 1st order ODE