

$$\vec{V}_A = c \hat{y}$$

$$\vec{V}_B = ?$$

$\hat{i} : \text{scalar}$
 $\hat{j} : \theta a$

$$\vec{V}_B = \vec{V}_A + \dot{\theta} \hat{z} \times \vec{r}_{A \rightarrow B}$$

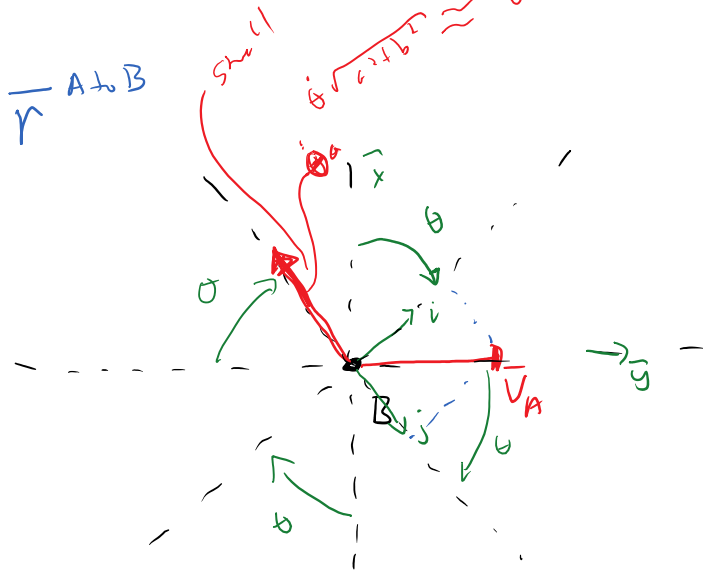
$$\vec{V}_B = c \sin \theta \hat{i} + c \cos \theta \hat{j}$$

assumptions
 $a \gg b$

θ is small

$$\hat{i}: c \sin \theta \approx c \theta$$

$$\hat{j}: c \cos \theta - \theta a \approx c - \theta a$$



$$\dot{r} + A r = 0 \quad A = \frac{m b U}{I_2 + m b^2}$$

$$m, b, I_2 > 0$$

$$\dot{r} = -A r$$

U can be + or -

$$r(t) = R e^{st}$$

$$\dot{r}(t) = R s e^{st}$$

$$R s e^{st} = -A R e^{st}$$

$$(s + A) R e^{st} = 0$$

R can be zero \Rightarrow trivial
 e^{st} cannot be zero not interesting!

$s + A = 0 \Rightarrow$ solution!

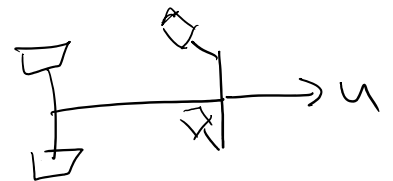
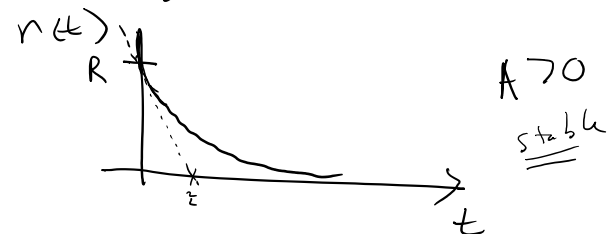
$$s = -A$$

eigenvalue s

$$A = \frac{m b U}{I_2 + m b^2}$$

$$r(t) = R e^{-A t}$$

solution



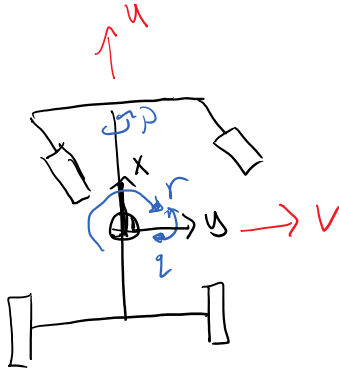
τ : time to decrease or increase to $1/e$

$$r(t) = R e^{\frac{t}{\tau}}$$

$$\tau = \frac{1}{|A|} = \frac{I_2 + m b^2}{m b |U|}$$

Body Fixed Coordinates

Inertial frame: doesn't accelerate, constant velocity or zero velocity



x, y, z body fixed coordinate system

x : long. y : lat z : vertical

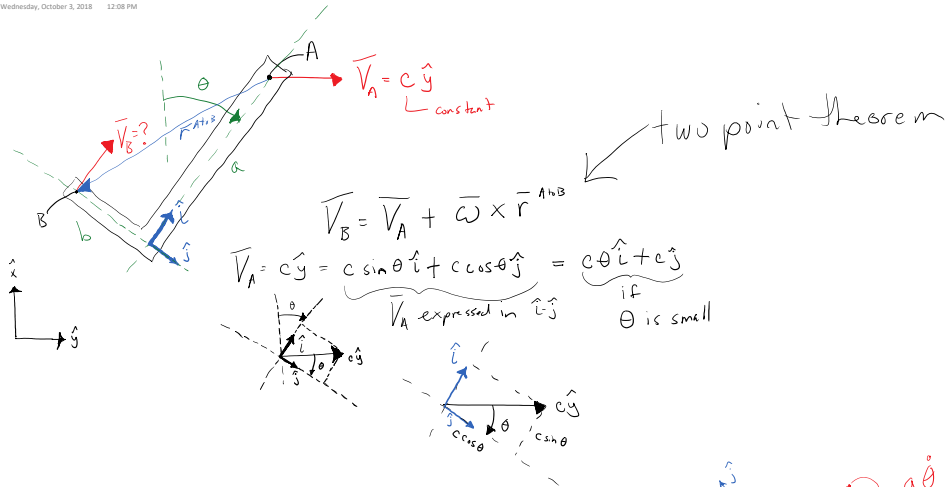
6 DoF body

u, v, w : velocity components

p, q, r : angular velocity components

X, Y, Z : components of force

L, M, N : components of moment



$$\vec{V}_B = \vec{V}_A + \vec{\omega} \times \vec{r}^{A \rightarrow B}$$

$$\vec{V}_A = c\dot{\theta}\hat{j} = c\sin\theta\dot{\theta}\hat{i} + c\cos\theta\dot{\theta}\hat{j} = c\dot{\theta}\hat{i} + c\dot{\theta}\hat{j}$$

\vec{V}_A expressed in \hat{i}, \hat{j} if θ is small

$$\vec{\omega} \times \vec{r}^{A \rightarrow B} = \dot{\theta}\hat{k} \times (-a\hat{i} - b\hat{j})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \dot{\theta} \\ -a & -b & 0 \end{vmatrix} = b\dot{\theta}\hat{i} - a\dot{\theta}\hat{j} + 0\hat{k}$$

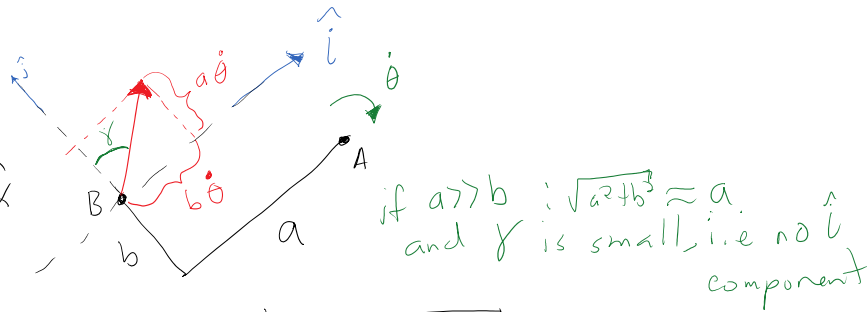
note the simplicity when expressed in the $\hat{i}, \hat{j}, \hat{k}$ frame

$$\vec{V}_B = (c\sin\theta + b\dot{\theta})\hat{i} + (c\cos\theta - a\dot{\theta})\hat{j}$$

Non-linear vel.

$$\vec{V}_B = (c\dot{\theta} + b\dot{\theta})\hat{i} + (c - a\dot{\theta})\hat{j} \rightarrow \text{assume } \theta \text{ is small, linear vel.}$$

$$\vec{V}_B = c\dot{\theta}\hat{i} + (c - a\dot{\theta})\hat{j} \text{ if } a \gg b \text{ and } \theta \text{ is small}$$



$$|\vec{\omega} \times \vec{r}^{A \rightarrow B}| = \dot{\theta}\sqrt{a^2 + b^2}$$