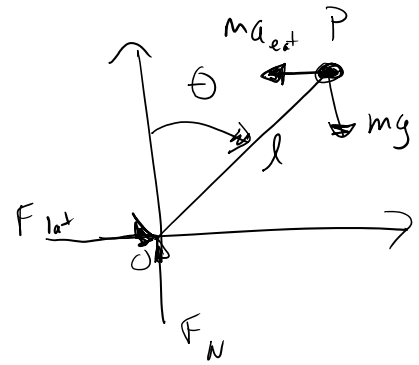


$$a = \frac{v^2}{r}$$



$$\sum F_r = 0$$

$$-mg \sin \theta + m a_{\text{cent}} \cos \theta = 0$$

$$a_{\text{cent}} = g \frac{\sin \theta}{\cos \theta} = g \tan \theta$$

① $x = \underline{X} e^{st}$

$$(a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n) \underline{X} e^{st} = 0$$

$\neq 0$ characteristic equation

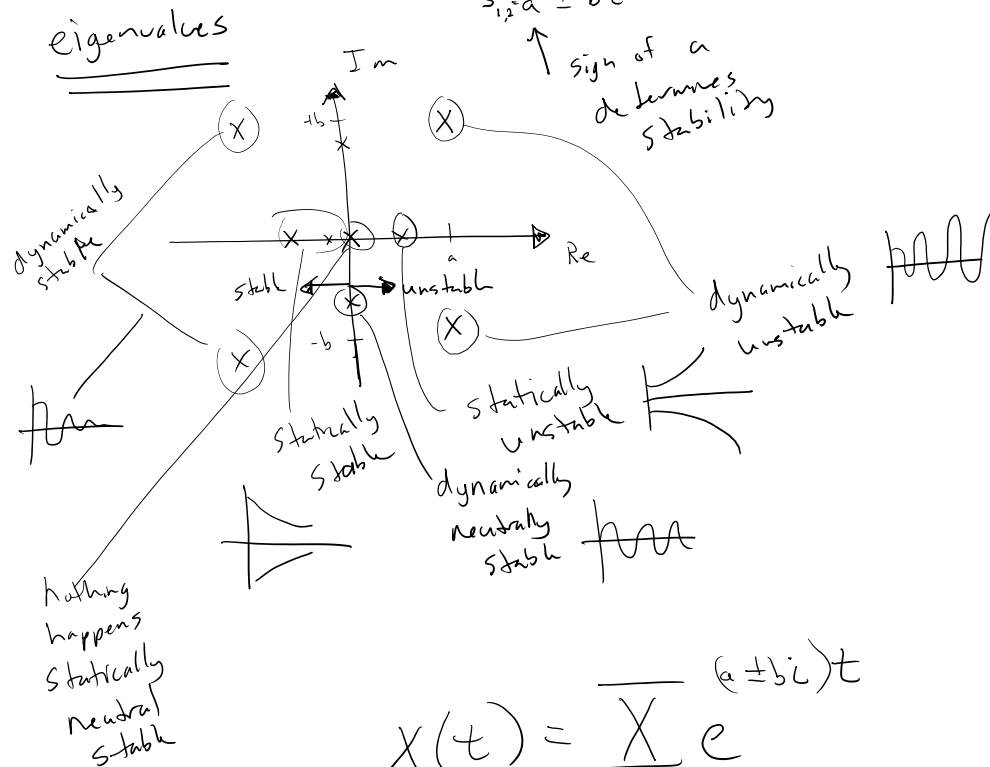
n values of s that can either complex or real

imag. solutions always come in complex

conjugate pairs $\Rightarrow a + bi \quad a - bi$

$$i = \sqrt{-1} \quad \text{pair} \quad s_{1,2} = a \pm bi$$

sign of a determines stability



$$x(t) = \underline{X} e^{(a \pm bi)t}$$

$$x(t) = A e^{at} \sin(bt + \phi)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

②

$$\dot{[x]} = [A][x]$$

$$[x] = [\underline{X}] e^{st}$$

$$[sI - A][\dot{x}] e^{st} = 0$$

$$\det [sI - A] = 0 \Rightarrow \text{characteristic eq.}$$

\Downarrow
gives polynomial in s

(3)

$$+ [C][\dot{x}] + [K][x] = 0$$

$$\det [Ms^2 + Cs + K] = 0$$

\Downarrow also gives char. eq.

ex

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - [A]$$

$$= \begin{bmatrix} s - a_{11} & -a_{12} \\ -a_{21} & s - a_{22} \end{bmatrix}$$

$$\det \begin{bmatrix} s - a_{11} & -a_{12} \\ -a_{21} & s - a_{22} \end{bmatrix} = 0$$

$$\underset{\substack{\uparrow \\ a_0}}{1}s^2 + \underbrace{(-a_{11} - a_{22})}_{a_1}s + \underbrace{(a_{11}a_{22} - a_{12}a_{21})}_{a_2} = 0$$

$$(3) [M][\ddot{x}] + [C][\dot{x}] + [K][x] = 0$$

$$\det \begin{bmatrix} m_{11}s^2 + c_{11}s + k_{11} & m_{12}s^2 + c_{12}s + k_{12} \\ m_{21}s^2 + c_{21}s + k_{21} & m_{22}s^2 + c_{22}s + k_{22} \end{bmatrix} = 0$$

$$\det \begin{bmatrix} m_{11}s^2 + c_{11}s + k_{11} & m_{12}s^2 + c_{12}s + k_{12} \\ m_{21}s^2 + c_{12}s + k_{21} & m_{22}s^2 + c_{22}s + k_{22} \end{bmatrix} = 0$$

Routh's Stability Criteria

Know if ^{real part of the} positive root to char. eq. without solving for that root

Procedure

① $a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n = 0$

② If any coefficient, a_n , is negative or zero and at least one other positive coefficient is present, the system is unstable!

For stability \Rightarrow necessary that all coeff be positive.

③ If all coeff are positive build this table:

s^n	a_0	a_2	a_4	a_6	...	$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$
s^{n-1}	a_1	a_3	a_5	a_7	...	$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$
s^{n-2}	b_1	b_2	b_3	b_4	...	$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$
s^{n-3}	c_1	c_2	c_3	c_4	...	$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$
\vdots						\vdots
s^2	e_1	e_2				
s^1	f_1					
s^0	g_1					

Criteria

of roots with positive real parts
is equal to # changes in sign
of the coefficients of the first column
of the table.