



$$m\ddot{x} + b\dot{x} + kx = 0$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 0$$

↑
damping
ratio

↑
nat. freq

$$\omega_n = \sqrt{\frac{C_2(a+b)}{I_c + ma^2}}$$

$$\zeta =$$

$$\frac{C_2(a+b)^2}{2u\sqrt{I_c + ma^2}\sqrt{C_2(a+b)}}$$

check this

if $m, b, \text{ or } k < 0 \Rightarrow$ unstable

$\zeta < 0 \Rightarrow$ unstable

$0 < \zeta < 1 \Rightarrow$ damped oscillation

$\zeta = 1 \Rightarrow$ critically damped (no osc.)

$\zeta > 1 \Rightarrow$ overdamped (doesn't oscillate)



$$x = A e^{-\zeta\omega_n t} [\sin(\omega_d t + \phi)]$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

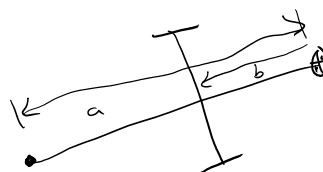
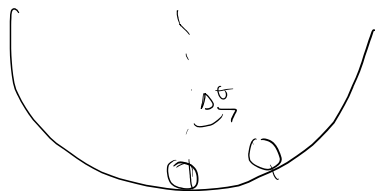
if all parameters are positive, $\zeta > 0 \Rightarrow$ stable behavior

if $u < 0$

$$(ma^2 + I_c)\ddot{\theta} + C_2 \frac{(a+b)^2}{u} \dot{\theta} - C_2(a+b)\theta = 0$$

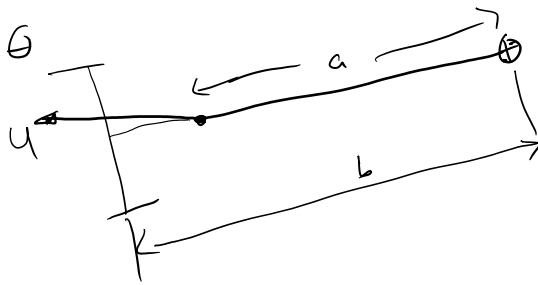
$u > 0$

$$= \underbrace{(a+b)\theta} - \underbrace{C_2 \frac{(a+b)^2}{u} \dot{\theta}} = \underline{\underline{\theta}}$$

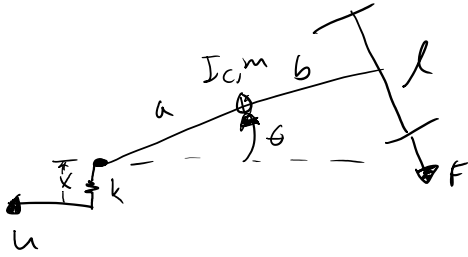


$$u > 0 \quad + C_2 (a+b) \theta$$

$$u < 0 \quad - C_3 (a+b) \theta$$



TWO DOF TRAILER



Parameters : $U, a, b, m, I_c, l, c_2, k$

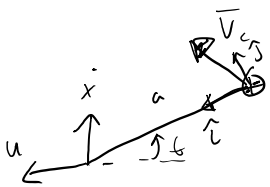
Variables : F, θ, x

generalized coordinates
minimal set of coordinates
that fully describe
the system's configuration
 $\# \text{ G.C.s.} = \# \text{ D.o.F}$

$$\frac{d}{dt} \frac{\partial T^*}{\partial \dot{\theta}} - \frac{\partial T^*}{\partial \theta} + \frac{\partial V}{\partial \theta} = \underline{\quad}_{\theta}$$

$$\frac{d}{dt} \frac{\partial T^*}{\partial \dot{x}} - \frac{\partial T^*}{\partial x} + \frac{\partial V}{\partial x} = \underline{\quad}_x$$

$T^* \quad V_c^2, \omega^2$



$$\vec{V}_c = (U \cos \theta - \dot{x} \sin \theta) \hat{i} + (U \sin \theta + \dot{x} \cos \theta) \hat{j} + a \dot{\theta} \hat{j}$$

$$V_c^2 = (U \cos \theta - \dot{x} \sin \theta)^2 + (U \sin \theta + \dot{x} \cos \theta + a \dot{\theta})^2$$

$$V_c^2 = U^2 + \dot{x}^2 + a^2 \dot{\theta}^2 + 2Ua \dot{\theta} \sin \theta + 2\dot{x}a \dot{\theta} \cos \theta$$