

HW 3.6, 4.4, 5.10 Due Monday  
use lagrange method

$$T^* = \frac{1}{2}m(u^2 + \dot{x}^2 + \dot{\theta}^2 + 2u\dot{\theta}\sin\theta + 2\dot{x}\dot{\theta}\cos\theta) + \frac{1}{2}I_c\dot{\theta}^2$$

### Potential Energy



$$F = kx$$

$$V_{\text{spring}} = \frac{1}{2}kx^2 = V$$

### Generalized Forces

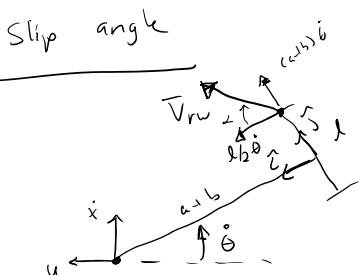
$$\sum_{\theta} \delta W = \sum_{\theta} \delta\theta$$

$$\sum_x \delta W = \sum_x \delta x$$

$$\sum_{\theta} \delta W = -F(a+b) \underbrace{\delta\theta}_{\sum_{\theta}}$$

$$\sum_x \delta W = -F \underbrace{\delta x}_{\sum_x}$$

$$F = C_x \propto$$



small perturbation about the basic motion

$$\theta = 0, x = 0$$

$x \neq 0$  & small

$\dot{x} \neq 0$  & small

$$\vec{v}_{rw} = (u \cos\theta - \dot{x} \sin\theta) \hat{i} + (u \sin\theta + \dot{x} \cos\theta) \hat{j} + \frac{\ell}{2} \dot{\theta} \hat{i} + (\alpha + b) \dot{\theta} \hat{j}$$

$$\omega = \frac{\dot{\theta}}{\ell} = \frac{u \sin\theta + \dot{x} \cos\theta + (\alpha + b) \dot{\theta}}{u \cos\theta - \dot{x} \sin\theta + \frac{\ell}{2} \dot{\theta}}$$

$$\omega = \frac{u \theta + \dot{x} + (\alpha + b) \dot{\theta}}{u - \dot{x} \theta + \frac{\ell}{2} \dot{\theta}} \quad \text{UT77 } \frac{1}{2} \dot{\theta}$$

$$\omega = \theta + \frac{(\alpha + b) \dot{\theta}}{u} + \frac{\dot{x}}{u}$$

Write EOM

1 .. . n-4

]. take d-

Write EoM

$$\frac{2T^*}{2\dot{\theta}} = m a^2 \ddot{\theta} + m u \dot{\theta} \sin \theta + m \dot{x} a \cos \theta + I_c \ddot{\theta}$$

$$\frac{2T^*}{2\dot{x}} = m u \dot{\theta} \cos \theta - m \dot{x} a \dot{\theta} \sin \theta$$

$$\frac{2V}{2\theta} = 0$$

$$\underline{\underline{\Sigma}_{\theta}} = -C_2 \left( \theta + \frac{(a+b)}{u} \dot{\theta} + \frac{\dot{x}}{u} \right) (a+b)$$

$\underbrace{\qquad\qquad\qquad}_{F}$

$$\underline{\underline{\Sigma}_x} = -C_2 \left( \theta + \frac{(a+b)}{u} \dot{\theta} + \frac{\dot{x}}{u} \right)$$

$$T^* = \frac{1}{2} m \left( u^2 + \dot{x}^2 + a^2 \dot{\theta}^2 + \underline{\underline{2u\dot{\theta}\sin\theta + 2\dot{x}a\dot{\theta}\cos\theta}} \right) + \frac{1}{2} I_c \dot{\theta}^2$$

$$V = \frac{1}{2} k x^2$$

$$\theta \quad (1) \quad m a^2 \ddot{\theta} + m \dot{x} a + I_c \ddot{\theta} + C_2 \left( \theta + \frac{(a+b)}{u} \dot{\theta} + \frac{\dot{x}}{u} \right) (a+b) = 0$$

$$x \quad (2) \quad m \ddot{x} + m \dot{\theta} + kx + C_2 \theta + \frac{C_2(a+b)}{u} \dot{\theta} + \frac{C_2}{u} \dot{x} = 0$$

2 2<sup>nd</sup> order ODEs  $\Rightarrow$  4<sup>th</sup> order coupled system

2X #DoF = order of system

$$\mathcal{M} \begin{bmatrix} \ddot{\theta} \\ \ddot{x} \end{bmatrix} + C \begin{bmatrix} \dot{\theta} \\ \dot{x} \end{bmatrix} + K \begin{bmatrix} \theta \\ x \end{bmatrix} = 0$$

$$\mathcal{M} = \begin{bmatrix} I_c + m a^2 & ma \\ ma & m \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{C_2(a+b)^2}{u} & \frac{C_2(a+b)}{u} \\ \frac{C_2(a+b)}{u} & \frac{C_2}{u} \end{bmatrix}$$

$$K = \begin{bmatrix} C_2(a+b) & 0 \\ 0 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} c_2(a+b) & 0 \\ 0 & K \end{bmatrix}$$

$$\det [Ms^2 + Cs + K] = 0$$

4th order polynomial

$$\begin{aligned} \cancel{c \omega \stackrel{eq}{=} } \quad & m I_c s^4 + (I_c + mb^2)(c_2/u)s^3 + [(I_c + ma^2)k + mb c_2]s^2 \\ & + [c_2(a+b)^2 k/u]s + c_2(a+b)k = 0 \end{aligned}$$

$$m, I_c, k, c_2, u > 0$$

$$(I_c + ma^2)k + mb c_2 < 0 \rightarrow \text{unstable} \quad b < \frac{(I_c + ma^2)k}{mc_2}$$

$$c_2(a+b)k < 0 \rightarrow \text{unstable} \quad b < -a$$