

$$\left. \frac{d\vec{v}}{dt} \right|_{\text{inertial frame}} = \left. \frac{d\vec{v}}{dt} \right|_{\text{moving frame}} + \vec{\omega} \times \vec{v}$$

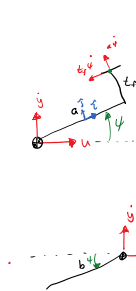
moving frame in the inertial frame

$$\Sigma F_x = m\ddot{x}$$

$$\Sigma F_y = m\ddot{y}$$

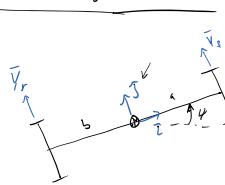
$$\Sigma T_c = I_z \ddot{\psi}$$

Time forces



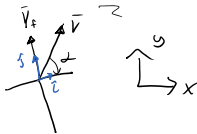
$$\begin{aligned} \vec{v}_{fx} &= (u \cos \psi + y \sin \psi - r \dot{\psi}) \hat{i} + (-u \sin \psi + y \cos \psi + r \dot{\psi}) \hat{j} \\ \alpha_{fx} &\approx \frac{-u \sin \psi + y \cos \psi + r \dot{\psi}}{u \cos \psi + y \sin \psi - r \dot{\psi}} \\ &\approx \frac{-u \sin \psi + y \cos \psi}{u \cos \psi + y \sin \psi - r \dot{\psi}} \quad u \gg r \dot{\psi} \\ \alpha_{fx} &\approx \frac{y \cos \psi}{u} \psi = \psi \\ \alpha_{rx} &\approx \frac{y - b \dot{\psi}}{u} - \psi = \alpha_r \end{aligned}$$

Note on signs of force



\vec{v}_f and \vec{v}_r shown as + for + slip angles.
if direction (or sign) of \vec{v}_f is - \hat{j} is + corresponds to a positive slip angle

$$S_o \quad Y_f = -C_f \alpha_f \quad \text{and} \quad Y_r = -C_r \alpha_r$$



$$\Sigma F_x$$

$$\Sigma F_x = m\ddot{x} \Rightarrow -Y_f \sin \psi - Y_r \sin \psi = m\ddot{x}$$

$$\Sigma F_y = m\ddot{y} \Rightarrow Y_f \cos \psi + Y_r \cos \psi = m\ddot{y}$$

$$-C_f \alpha_f - C_r \alpha_r = m\ddot{y}$$

$$\Sigma T_c = I_z \ddot{\psi} \Rightarrow Y_f a - Y_r b = I_z \ddot{\psi}$$

$$-C_f \alpha_f a + C_r \alpha_r b = I_z \ddot{\psi}$$

Matrix Form \Rightarrow plug in α_f, α_r and find their coeffs.

$$\begin{bmatrix} m & 0 \\ 0 & I_z \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} (c_f + c_r)/u & (a c_f - b c_r)/u \\ (a c_f - b c_r)/u & (a^2 c_f + b^2 c_r)/u \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & -(c_f + c_r) \\ 0 & -(a c_f - b c_r) \end{bmatrix} \begin{bmatrix} y \\ \psi \end{bmatrix} = 0$$

Char. eq.

$$m I_z s^4 + \left[m (a^2 c_f + b^2 c_r) / u + I_z (c_f + c_r) / u \right] s^3 + \left[\frac{(c_r + c_f)(a^2 c_f + b^2 c_r)}{u^2} - \frac{(a c_f - b c_r)^2}{u^2} - m(a c_f - b c_r) \right] s^2 + 0 s^1 + 0 s^0 = 0$$

investigate for stability
 a_2

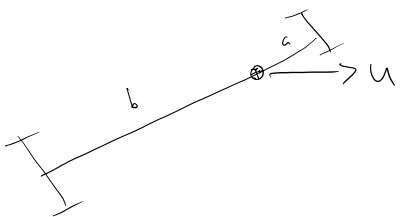
! divide through by $s^2 \Rightarrow$ a 2nd order polynomial

$a_2 > 0$ to be stable

$$(a+b)^2 c_f c_r / u^2 + m(b c_r - a c_f) > 0$$

if u is small $(a+b)^2 c_f c_r$ is large! \Rightarrow at slow speeds car is stable

if second term is positive \Rightarrow always stable if $b c_r > a c_f$ then second term is positive



understeer condition

$b c_r > a c_f$ understeer

if second term is negative then there is a speed at which the system can be unstable!

if $b c_r < a c_f$ then:

$$U_{crit}^2 = \frac{-(a+b)^2 c_f c_r}{m(b c_r - a c_f)}$$

above this speed \Rightarrow unstable

$b c_r < a c_f$ oversteer

~~oversteer!~~ oversteer!

generating too large of lateral forces at the front end