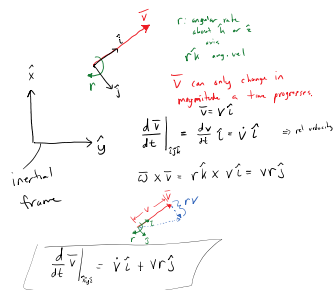
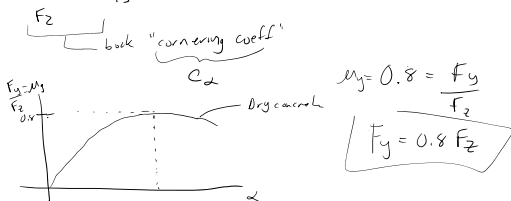


Why is  $\left. \frac{d\vec{v}}{dt} \right|_{\text{inertial frame}} = \left. \frac{d\vec{v}}{dt} \right|_{\text{rotating frame}} + \vec{\omega} \times \vec{v}$ ?



### Problem 4.3 badly worded

'It is stated in the text that if the maximum is 0.8 ...'



Some confusion about getting char. equation

$M\ddot{\vec{x}} + B\dot{\vec{x}} + K\vec{x} = 0$   $\vec{x}$  is  $n \times 1$  vector  
then  $M, B$ , and  $K$  are  $n \times n$  matrices

if  $n=2$   
 $M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$ ,  $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ ,  $K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$   $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\det[Ms^2 + Bs + K] = 0$$

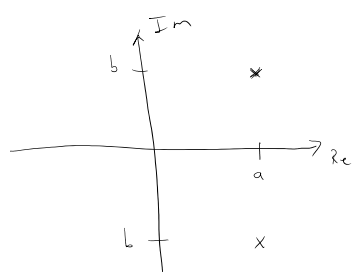
$$\det \begin{bmatrix} m_{11}s^2 + b_{11}s + k_{11} & m_{12}s^2 + b_{12}s + k_{12} \\ m_{21}s^2 + b_{21}s + k_{21} & m_{22}s^2 + b_{22}s + k_{22} \end{bmatrix} = 0$$

$$(m_{11}s^2 + b_{11}s + k_{11})(m_{22}s^2 + b_{22}s + k_{22}) - (m_{12}s^2 + b_{12}s + k_{12})(m_{21}s^2 + b_{21}s + k_{21}) = 0$$

$$m_{11}m_{22}s^4 + m_{11}b_{22}s^3 + m_{11}k_{22}s^2 + b_{11}m_{22}s^3 + b_{11}b_{22}s^2 + b_{11}k_{22}s + k_{11}m_{22}s^2 + k_{11}b_{22}s + k_{11}k_{22} - [ \dots ] = 0$$

two coordinates  $\Rightarrow x_1, x_2$   $\rightarrow$  fourth order polynomial  
 $2 \times 2$   $M, B, K$   $2n$

### Root locus



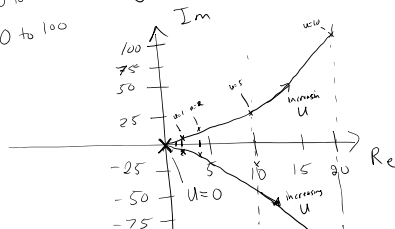
$$s = a \pm bj \quad j = \sqrt{-1}$$

$a, b$  can be functions of system parameters

$$s = f(u) \pm g(u)j$$

$$f = 2u \quad 0 \text{ to } 20 \text{ if vary } u \text{ from } 0 \text{ to } 10$$

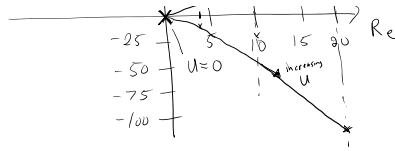
$$g = u^2 \quad 0 \text{ to } 100$$



### root locus

- vary single parameter
- plot complex root locations

- vary single parameter
- plot complex root locations



## "Generalized force" in Lagrange's Eq

$$F = ma$$

$$F = m l \ddot{\theta}$$

↑ units force N  
↑ Torque units N·m

$$\frac{d}{dt} \frac{\partial T^*}{\partial \dot{\theta}} - \frac{\partial T^*}{\partial \theta} + \frac{\partial V}{\partial \theta} = \underline{\underline{\quad \quad \quad}}_{\theta}$$

then equations results in a torque/moment balance

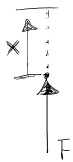
$$\frac{d}{dt} \frac{\partial T^*}{\partial \dot{x}} - \frac{\partial T^*}{\partial x} + \frac{\partial V}{\partial x} = \underline{\underline{\quad \quad \quad}}_x$$

units of N·m

generalized force

↳ force balance  $\Rightarrow N$

$$\underline{\underline{\quad \quad \quad}}_x = ?$$



$$F_x = W \Rightarrow \text{positive}$$

$$\underline{\underline{\quad \quad \quad}}_x = F$$



$$-F_x = W \Rightarrow \text{neg}$$

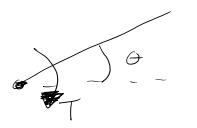
$$\underline{\underline{\quad \quad \quad}}_x = -F$$

$$\underline{\underline{\quad \quad \quad}}_{\theta} = ?$$



$$T l = W \Rightarrow \text{pos}$$

$$\underline{\underline{\quad \quad \quad}}_{\theta} = T$$



$$-T l = W \Rightarrow \text{neg}$$

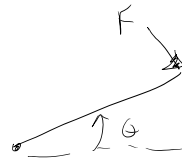
$$\underline{\underline{\quad \quad \quad}}_{\theta} = -T$$

$$\underline{\underline{\quad \quad \quad}}_{\theta} = ?$$



$$F l = W \Rightarrow \text{pos}$$

$$\underline{\underline{\quad \quad \quad}}_{\theta} = F l$$



$$-F l = W$$

$$\underline{\underline{\quad \quad \quad}}_{\theta} = -F l$$

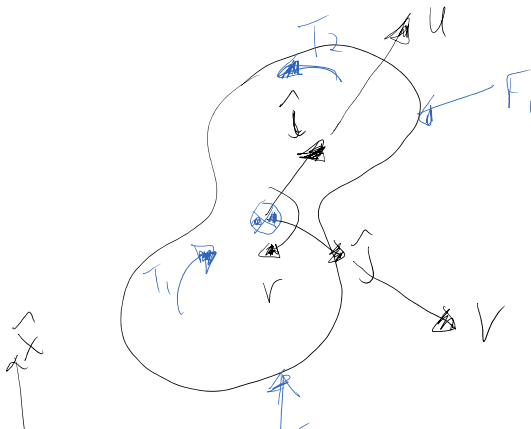
## Body fixed coordinates

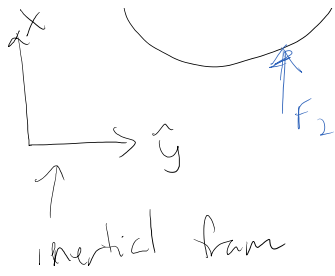
### Newton's 2nd Law

$$\sum \vec{F} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = m \begin{bmatrix} \ddot{u} - r\dot{v} \\ \ddot{v} + r\dot{u} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \sum F_i \\ \sum F_j \end{bmatrix} = m \begin{bmatrix} \ddot{u} - r\dot{v} \\ \ddot{v} + r\dot{u} \end{bmatrix}$$

linear motion





$v$

$$[\Sigma F_j] \quad (v + r\omega)$$

Euler's Equations

$$\Sigma T_z = N = I_{zz} \dot{\omega}$$