

Causality

Key thing: Is effort or flow the input at a port?

A function $g()$ and it's inverse $h()$
 where $h() = g^{-1}()$

$y = 2x^2$ $g(?) = 2(?)^2$

x is input and y is the output
 ↑
 independent variable ↑
 dependent variable

$x = \pm\sqrt{\frac{y}{2}}$ $h(?) = \pm\sqrt{\frac{(?)}{2}}$

y is input, x is output

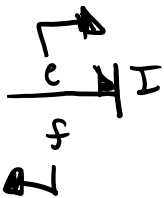
But what if:

$y = 3 \sin(z)$ and $z = x^3$

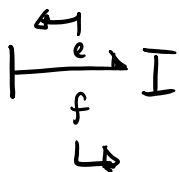
This takes two steps to go from input x to output y and we have an intermediate variable z .

Inertia

Integral Causality



Derivative



Step 1: integrate e to get p

Step 2: apply law

$p = \int e dt$

$f = g(p)$

Step 1: apply law to get p

Step 2: diff. p to get e

$p = h(f)$

dD

↳

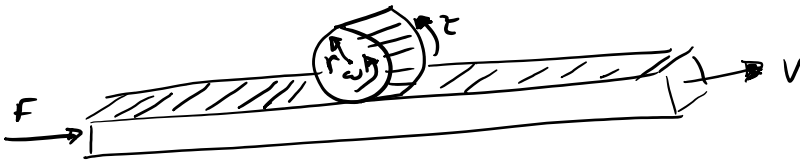
$$p = h(f)$$

$$\frac{dp}{dt} = e$$

Constitutive Laws

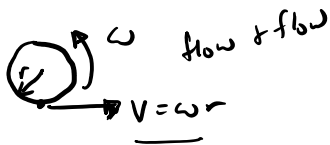
Given a basis system or element: pump, flywheel, a tank, etc.
 goal is to determine what variables change and write
 the mathematical relationship among the variables in
 form that corresponds to R, C, I, TF, GK, S_e, S_f, 1, 0
 parts.

3-14



r : constant

$F(t), \omega(t), \tau(t), V(t)$: not constant



$$\tau = rF$$

$$r\omega = v$$

fit



$$e_1 = m e_2$$

$$m f_1 = f_2$$

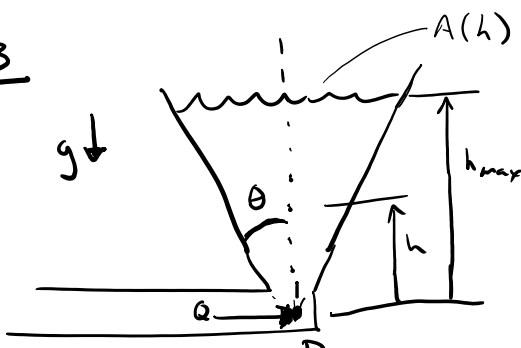
effort & effort

$$\underline{Fr = \tau}$$

$$\frac{\tau}{\omega} \xrightarrow{TF} \frac{F}{v}$$

$$\frac{e_1}{f_1} \xrightarrow{TF} \frac{e_2}{f_2}$$

3-3



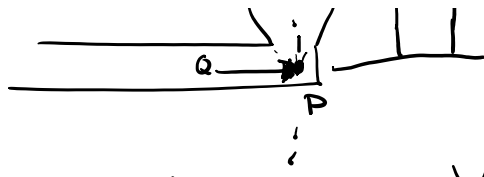
$$\frac{p}{a} \rightarrow c$$

$$P = g(v)$$

$$A = \pi r(h)^2$$

$$A = \pi (h \tan \theta)^2$$

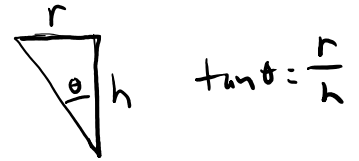
$$A = \pi \tan^2 \theta h^2$$



$$A = \pi \tan^2 \theta h^2$$

$$P = \rho g h_{\max}$$

$$V = \int_0^{h_{\max}} A(h) dh$$



$$V = \int_0^{h_{\max}} \pi \tan^2 \theta h^2 dh$$

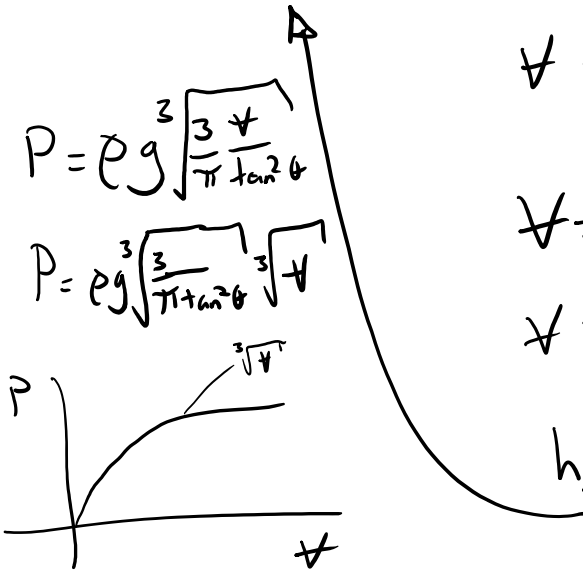
$$V = \pi \tan^2 \theta \int_0^{h_{\max}} h^2 dh$$

$$V = \pi \tan^2 \theta \frac{h_{\max}^3}{3}$$

$$h_{\max} = \sqrt[3]{\frac{3V}{\pi \tan^2 \theta}}$$

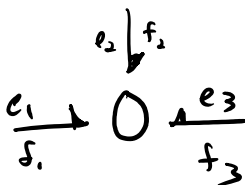
$$P = \rho g \sqrt[3]{\frac{3V}{\pi \tan^2 \theta}}$$

$$P = \rho g \sqrt[3]{\frac{3}{\pi \tan^2 \theta}} \sqrt[3]{V}$$



Sign Conventions

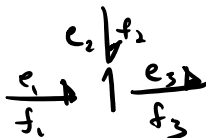
Always + power into C, I, R !



$$e_1 = e_2 = e_3$$

$$f_1 + f_2 + f_3 = 0$$

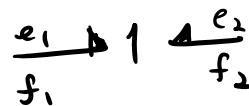
$$-f_1 - f_2 - f_3 = 0$$



$$f_1 = f_2 = f_3$$

$$e_3 = e_1 + e_2$$

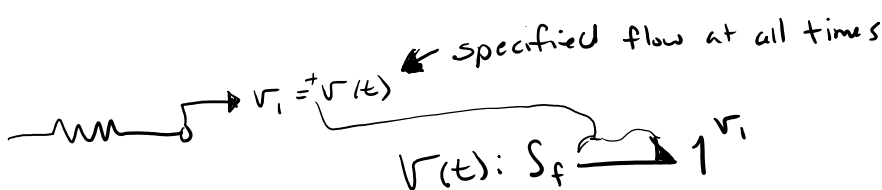
$$e_1 + e_2 - e_3 = 0$$

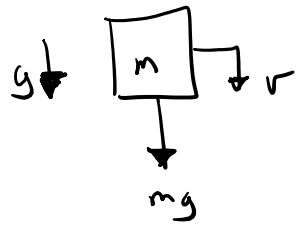
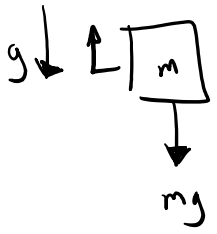


$$f_1 = f_2$$

$$e_1 + e_2 = 0$$

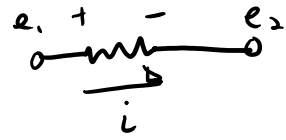
$$e_1 = -e_2$$



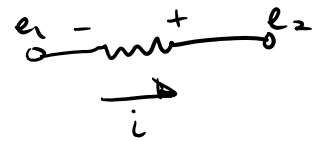


$mg: S_e \leftarrow 1^v$
 or
 $-mg: S_e \rightarrow 1^v$

$mg: S_e \rightarrow 1^v$
 or
 $-mg: S_e \leftarrow 1^v$



$e_1 \rightarrow 1 \leftarrow e_2$
 $e_1 - e_2 \downarrow$
 R



$e_1 \leftarrow 1 \rightarrow e_2$
 $e_2 - e_1 \downarrow$
 R