

- HW 7 Due Today
- Lab 4 due M Dec 2
- HW 8 due W Nov 27
- HW 9 due F Dec 6
- Lab 5 due M Dec 9

Chapter 5: Generating System Equations From Bond Graphs

State determined systems: future depends on present state and inputs

Can be described by a set of non-linear explicit first order differential equations:

$$\left. \begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, \dots, x_n, u_1, \dots, u_s, t) \\ &\vdots \\ \dot{x}_n &= f_n(x_1, x_2, \dots, x_n, u_1, \dots, u_s, t) \end{aligned} \right\} \dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, t)$$

vector form

$x \triangleq$ state variables, $u \triangleq$ input variable

Linear version of these equations:

$$\left. \begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n + b_{11}u_1 + \dots + b_{1s}u_s \\ &\vdots \\ \dot{x}_n &= a_{n1}x_1 + \dots + a_{nn}x_n + b_{n1}u_1 + \dots + b_{ns}u_s \end{aligned} \right\} \dot{\bar{x}} = A\bar{x} + B\bar{u}$$

vector/matrix form
"state space form"

A: state matrix

B: input matrix

$n \times n$

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$$

$n \times s$

$$B = \begin{bmatrix} b_{11} & \dots & b_{1s} \\ \vdots & & \vdots \\ b_{n1} & \dots & b_{ns} \end{bmatrix}$$

n : num states

s : num inputs

$$A = \begin{matrix} n \times n \\ \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \end{matrix}$$

$$B = \begin{matrix} n \times s \\ \begin{bmatrix} b_{11} & \dots & b_{1s} \\ \vdots & \ddots & \vdots \\ b_{n1} & \dots & b_{ns} \end{bmatrix} \end{matrix}$$

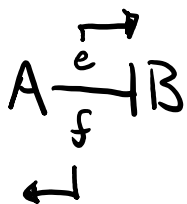
n : num states
 s : num inputs

$$\dot{\bar{x}} = A \bar{x} + B \bar{u}$$

Convert a Bond graph to one of these two forms!

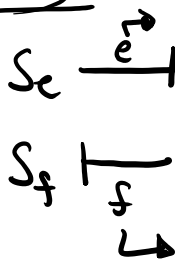
Causality Refresher

General



	A	B
input	f	e
output	e	f

Sources



always have this causality

Resistor



$$e = Rf$$

either
are
OK



$$f = \frac{1}{R}e$$

Capacitance

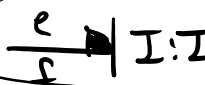
preferred

$$\left(\begin{matrix} e \\ f \end{matrix} \right) C:C \quad e = \frac{1}{C}q = \frac{1}{C} \int f dt \quad \text{integral}$$

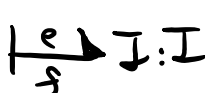
$$\rightarrow C:C \quad f = \frac{dq}{dt} = C \frac{de}{dt} \quad \text{derivative}$$

Inertia

preferred

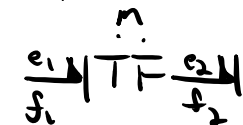


$$f = \frac{1}{I}p = \frac{1}{I} \int e dt \quad \text{integral}$$



$$e = \frac{dp}{dt} = I \frac{df}{dt} \quad \text{derivative}$$

Transformers



$$e_1 = m e_2$$

$$f_2 = m f_1$$

$$e_2 = \frac{1}{m} e_1$$

$$f_1 = \frac{1}{m} f_2$$

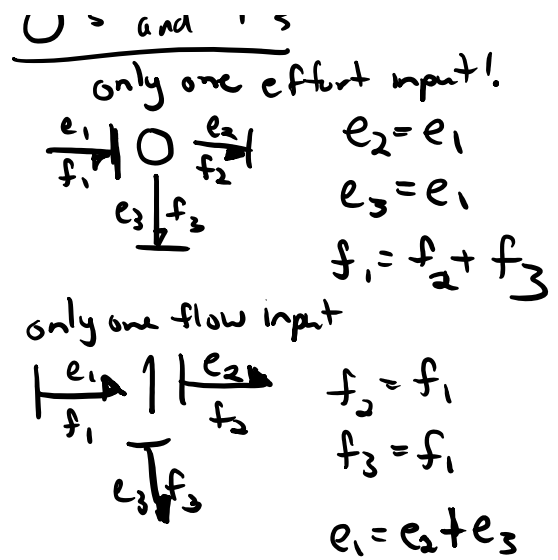
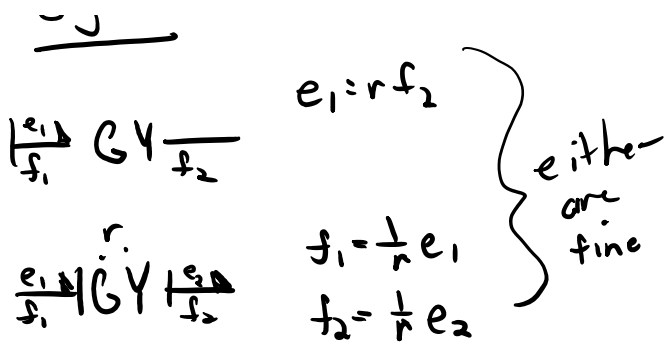
either
are
fine

Gyrator

$$e_1 = r f_2$$

0's and 1's

only one effort input!



Steps to assign causality to a system model

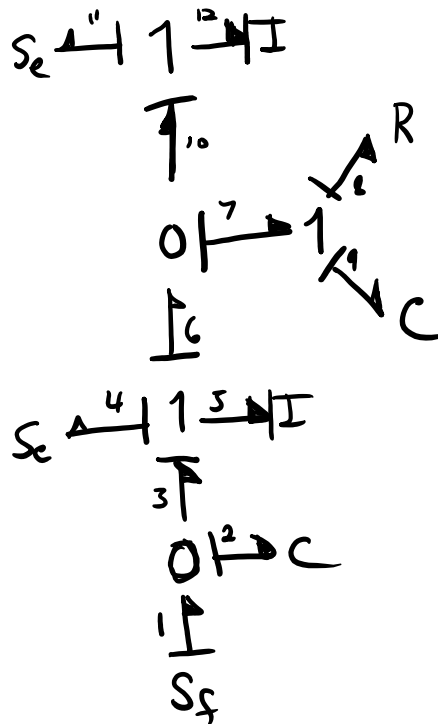
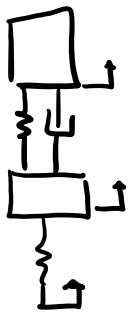
1. number the bonds
2. choose an S_e or S_f and assign causality
3. extend causality through TF, GY, O, I as far as possible
4. repeat 2 & 3 for all S_e or S_f
5. select an I or C and assign preferred integral causality (if possible)
6. extend causality through TF, GY, O, I as far as possible
7. repeat 5 & 6 for I and C
8. it is common that causality is complete, check for completeness and stop, or if not move on to 9.
9. choose any R and assign an arbitrary causality
10. extend causality through TF, GY, O, I
11. repeat 9 & 10 for all R ports
12. check for completeness, stop, or move on

13. choose any remaining bond without causality and assign an arbitrary mark

14. extend ...

15. repeat 13 & 14 until complete (all bonds have causal stroke)

Example 1



Example 2

