

- HW # 8 Due Wednesday
- Lab 4 Due Monday
- NO OH Wednesday by Jason, (can come 3-4 pm tomorrow (Tuesday))
- No OH Mon Dec 2 by Kevin
- Ask questions on canvas discussions

Finish problem from last time

$$\textcircled{1} \quad \dot{q}_4 = \frac{1}{R_2} \left( s_{e1} - \frac{q_4}{C_4} \right) - \frac{p_7}{I_7}$$

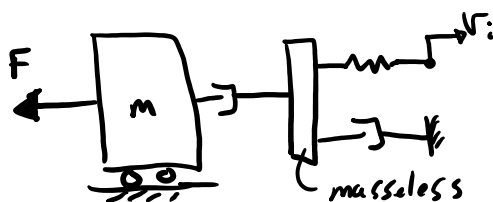
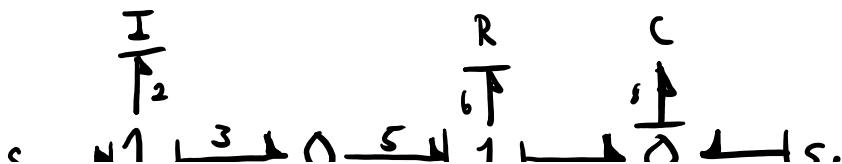
$$\textcircled{2} \quad \dot{p}_7 = \frac{q_4}{C_4} - R_6 \left( \frac{p_7}{I_7} \right) - \frac{q_8}{C_8}$$

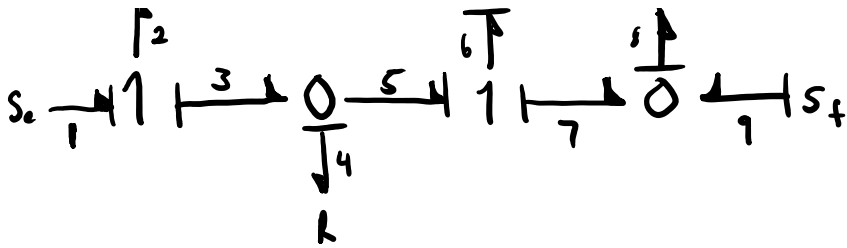
$$\textcircled{3} \quad \dot{q}_8 = \frac{p_7}{I_7}$$

Put in state space form:  $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$

$$\begin{matrix} 3 \times 1 \\ \begin{bmatrix} \dot{q}_4 \\ \dot{p}_7 \\ \dot{q}_8 \end{bmatrix} \end{matrix} = \begin{matrix} 3 \times 3 \\ \begin{bmatrix} -\frac{1}{R_2 C_4} & -\frac{1}{I_7} & 0 \\ \frac{1}{C_4} & -\frac{R_6}{I_7} & -\frac{1}{C_8} \\ 0 & \frac{1}{I_7} & 0 \end{bmatrix} \end{matrix} \begin{matrix} 3 \times 1 \\ \begin{bmatrix} q_4 \\ p_7 \\ q_8 \end{bmatrix} \end{matrix} + \begin{matrix} 3 \times 1 & 1 \times 1 \\ \begin{bmatrix} \frac{1}{R_2} \\ 0 \\ 0 \end{bmatrix} \end{matrix} [s_{e1}]$$

### Dealing with Algebraic Loops





Inputs:  $S_{e_1}, S_{e_8}$

States:  $P_2, q_8$

Causality of one  $R$  was arbitrarily selected. This is the result of a modeling assumption and causes an algebraic loop.

$$\dot{P}_2 = e_2 = S_{e_1} - e_3 = S_{e_1} - e_4 = S_{e_1} - R_4 f_4 = S_{e_1} - R_4 (f_3 - f_5)$$

$$\dot{P}_2 = S_{e_1} - R_4 \left( \frac{P_2}{I_2} - f_6 \right) = S_{e_1} - R_4 \left( \frac{P_2}{I_2} - \frac{e_6}{R_C} \right) = S_{e_1} - R_4 \left( \frac{P_2}{I_2} - \frac{e_5 - e_7}{R_C} \right)$$

$$\dot{P}_2 = S_{e_1} - R_4 \left( \frac{P_2}{I_2} - \frac{e_4 - e_7}{R_C} \right) \quad \leftarrow e_4 \text{ appears in equation again} \rightarrow \text{loop!}$$

$$\dot{P}_2 = S_{e_1} - R_4 \left( \frac{P_2}{I_2} - \frac{e_4 - \frac{q_8}{C_g}}{R_C} \right)$$

=  $e_4$

$$e_4 = R_4 \left( \frac{P_2}{I_2} - \frac{e_4 - \frac{q_8}{C_g}}{R_C} \right) \quad \text{solve for } e_4$$

$$e_4 + \frac{R_4}{R_C} e_4 = \frac{R_4}{I_2} P_2 + \frac{R_4}{R_C C_g} q_8$$

$$e_4 = \frac{\frac{R_4}{I_2} P_2 + \frac{R_4}{R_C C_g} q_8}{1 + \frac{R_4}{R_C}}$$

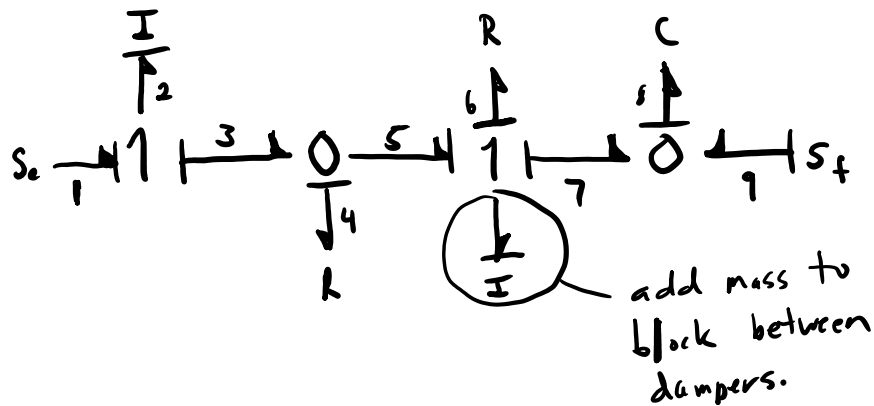
$$1 + \frac{R_4}{R_C}$$

So

$$\dot{P}_2 = S_{e_1} - \frac{R_4}{I_2} P_2 + \frac{R_4}{R_C C_g} q_8$$

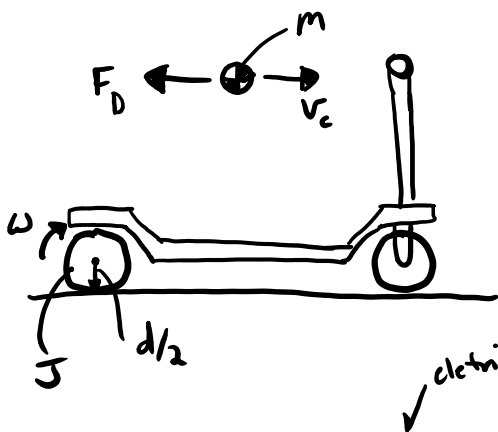
$$\dot{P}_2 = \frac{S e_1 - \frac{R_4}{I_2} P_2 + \frac{R_4}{R C C_0} v_8}{1 + \frac{R_4}{R_6}}$$

Algebraic loops can also be avoided by adding an  $I$  or  $C$  to a model (but there is 1 more state equation to track).



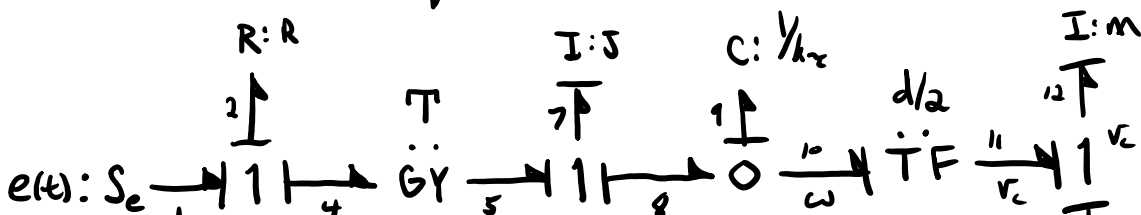
The new  $I$  forces the causality and no arbitrary  $R$  is chosen, thus no loop.

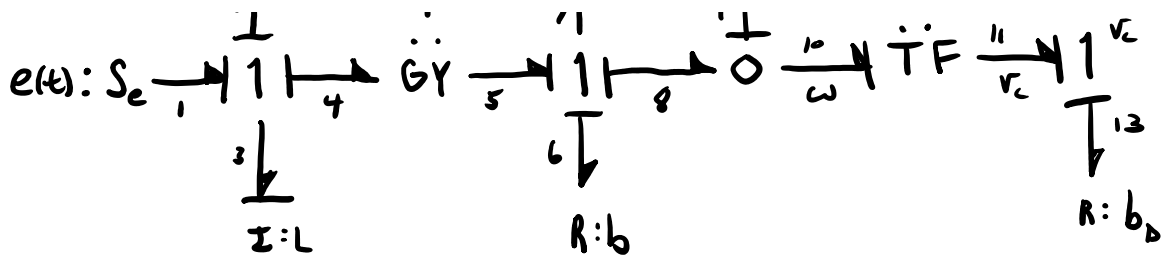
### Example: Electric Scooter



Inputs:  $S e_1$

States:  $P_3, P_7, q_9, P_{12}$





$$\dot{P}_3 = S_{e_1} - R_2 \frac{P_3}{I_3} - T \frac{P_7}{I_7} \xrightarrow{\text{use constants}} \dot{P}_3 = e(t) - \frac{R}{L} P_3 - T \frac{P_7}{J} \quad (1)$$

$$\dot{P}_7 = -R_6 \frac{P_7}{I_7} - \frac{q_9}{c_1} + T \frac{P_3}{I_2} = \dot{P}_7 = -\frac{b}{J} P_7 - k_T q_9 + \frac{T}{L} P_3 \quad (2)$$

$$\dot{q}_9 = \frac{1}{I_7} P_7 - \frac{2}{d} \frac{P_{12}}{I_{12}} \Rightarrow \dot{q}_9 = \frac{1}{J} P_7 - \frac{2}{dm} P_{12} \quad (3)$$

$$\dot{P}_{12} = \frac{2}{d} \frac{q_9}{c_1} - R_{13} \frac{P_{12}}{I_{12}} \Rightarrow \dot{P}_{12} = \frac{2k_T}{d} q_9 - \frac{b_0}{m} P_{12} \quad (4)$$

## State Space

$$\begin{bmatrix} \dot{P}_3 \\ \dot{P}_7 \\ \dot{q}_9 \\ \dot{P}_{12} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -T & 0 & 0 \\ \frac{T}{L} & -\frac{b}{J} & -k_T & 0 \\ 0 & \frac{1}{J} & 0 & -\frac{2}{dm} \\ 0 & 0 & \frac{2k_T}{d} & -\frac{b_0}{m} \end{bmatrix} \begin{bmatrix} P_3 \\ P_7 \\ q_9 \\ P_{12} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} [e(t)]$$

Outputs (in state space form):

$$\bar{y} = C \bar{v} + D \bar{u} \quad C: \text{output matrix}$$

$$\bar{y} = C \bar{x} + D \bar{u}$$

C: output matrix

D: feed forward matrix

some linear outputs:

$$v_c = P_{12}/m \quad \omega = f_{10} = \frac{2}{d} f_{11} = \frac{2}{d} \frac{P_{12}}{m}$$

$$\omega_m = P_7/5$$

motor

$$\bar{y} = \begin{matrix} 3 \times 1 \\ \left[ \begin{array}{c} v_c \\ \omega_m \\ \omega \end{array} \right] \end{matrix} = \begin{matrix} 3 \times 4 \\ \left[ \begin{array}{cccc} 0 & 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{5} & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{dm} \end{array} \right] \end{matrix} \begin{matrix} 4 \times 1 \\ \left[ \begin{array}{c} P_3 \\ P_7 \\ 2g \\ P_{12} \end{array} \right] \end{matrix} + \begin{matrix} 3 \times 1 & 1 \times 1 \\ \left[ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \end{matrix} [e(t)]$$

C D