

Derivative Causality

$\dot{\bar{X}} = \bar{f}(\bar{x}, t) \Rightarrow$ explicit ODE (all derivatives on left hand side)

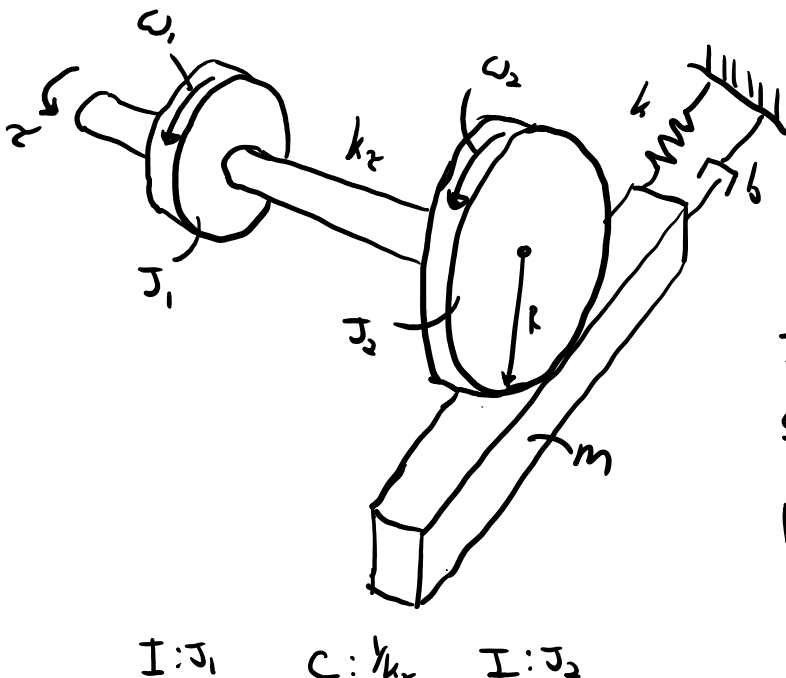
$\dot{\bar{X}} = \bar{f}(\bar{x}, \dot{\bar{x}}, t) \Rightarrow$ implicit ODE (derivatives on right and left hand sides)

For linear systems, implicit can be turned into explicit with some algebra.

For non-linear systems, implicit may be turned into explicit with algebra, but not always. If not possible, this leads to a set of differential algebraic equations (DAE). These are possible to solve numerically but require special solvers.

Derivative causality leads to implicit ODEs.

Example

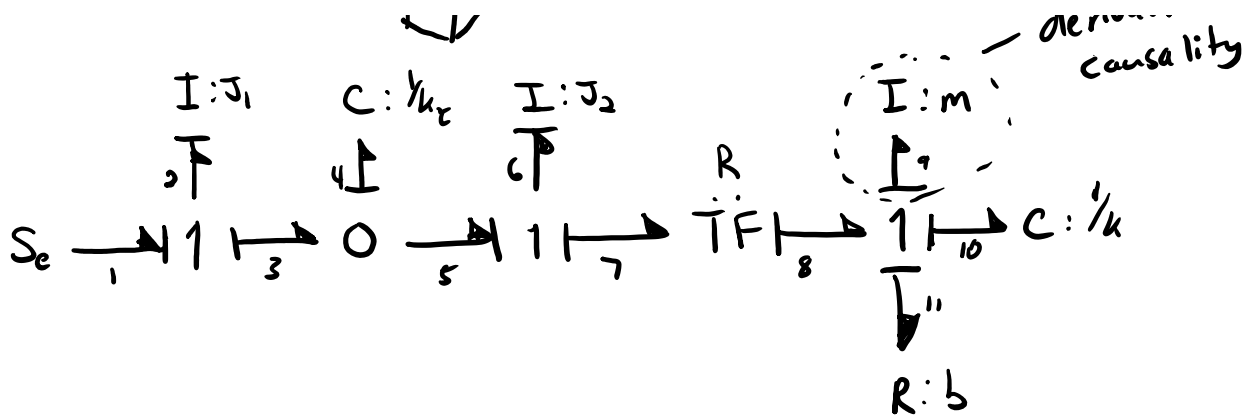


Inputs: τ, b

States: $p_2, \dot{q}_2, p_6, \dot{q}_6$

p_9 is not a state!

derivative causality



Start with the derivative causality bonds.

Write the constitutive law for the derivative causality bond.

$$P_q = I_q \dot{q}_q \quad (\text{don't write } \dot{p}_q = e_q \text{ like you do for states})$$

now follow causality

$$P_q = I_q \dot{q}_q = I_q R \dot{q}_7 = I_q R \dot{q}_C = I_q R \frac{P_C}{I_C}$$

(A) $P_q = \frac{I_q R}{I_C} P_C \Rightarrow$ this is an algebraic relationship between two states that arises from the derivative causality.

Take the ^{time} derivative of (A)

$$(B) \dot{P}_q = \frac{I_q R}{I_C} \dot{P}_C$$

Do this for all derivative causality bonds. You will end up with an equation for each derivative causality bond. You can then solve these equations for all the time derivatives of the non-state \dot{p} 's and \dot{q} 's. These can be used to substitute into the state equations when you encounter a non-state p or q .

So, do state equations as normal now:

$$\textcircled{1} \dot{p}_2 = e_2 = s_{c1} - e_3 = s_{c1} - \frac{q_4}{C_4}$$

$$\textcircled{2} \dot{q}_4 = f_4 = f_3 - f_5 = \frac{p_2}{I_2} - \frac{p_6}{I_6}$$

$$\textcircled{3} \dot{p}_6 = e_6 = e_5 - e_7 = \frac{q_4}{C_4} - R e_8 = \frac{q_4}{C_4} - R(e_9 + e_{10} + e_{11})$$

$$\dot{p}_6 = \frac{q_4}{C_4} - R \left(\dot{p}_9 + \frac{q_{10}}{C_{10}} + R_{11} f_{11} \right)$$

don't use
constitutive
law, replace
eq with \dot{p}_9

$$\dot{p}_6 = \frac{q_4}{C_4} - R \left(\dot{p}_9 + \frac{q_{10}}{C_{10}} + R_{11} R \frac{p_6}{I_6} \right)$$

now substitute in \textcircled{B} for \dot{p}_9

$$\dot{p}_6 = \frac{q_4}{C_4} - R \left(\frac{I_9 R}{I_6} \dot{p}_6 + \frac{q_{10}}{C_{10}} + R_{11} R \frac{p_6}{I_6} \right)$$

└ \dot{p}_6 on both sides

solve for p_6 to get in explicit form

$$\textcircled{3} \dot{p}_6 = \frac{\frac{q_4}{C_4} - R \left(\frac{q_{10}}{C_{10}} + R_{11} R \frac{p_6}{I_6} \right)}{1 + \frac{I_9 R^2}{I_6}}$$

⇐ explicit state equation
with \dot{p}_9 eliminated

$$\textcircled{4} \dot{q}_{10} = f_{10} = R f_7 = R \frac{p_6}{I_6}$$