

- Lab 4 Due today
- Lab 5 Due Monday
- HW 9 Due Friday
- Final Thursday, Dec 12 3:30 PM

## Analysis of Linear Systems

State space form of a set of linear ordinary differential equations.

$$\dot{\bar{x}} = A\bar{x} + B\bar{u}$$

$$\bar{y} = C\bar{x} + D\bar{u}$$

A: state matrix

B: input matrix

C: output matrix

D: feed forward matrix

We've been solving these by numerically integrating  $\dot{\bar{x}}(t)$  to find  $\bar{x}(t)$ .

$$\bar{x} = \int_{t_0}^{t_f} \dot{\bar{x}} dt = \int_{t_0}^{t_f} (A\bar{x} + B\bar{u}) dt$$

If the system is linear we can also find  $\bar{x}$  using analytical methods.

The first step is to find the solution to  $\dot{\bar{x}} = A\bar{x}$ , the homogeneous solution. Set  $\bar{u} = 0$  to work with non-forced

System.

Similar to single linear ODE's a solution of this form works:

$$\bar{x}(t) = \bar{X} e^{st} \quad \dot{\bar{x}} = \bar{X} s e^{st}$$
$$\dot{\bar{x}} = A\bar{x} \Rightarrow \bar{X} s e^{st} = A\bar{X} e^{st} \Rightarrow \bar{X} e^{st} [sI - A] = 0$$

identity matrix

$\bar{X} = 0 \Rightarrow$  trivial solution

$e^{st} = 0 \Rightarrow$  never true

$\bar{X} [sI - A] = 0 \Rightarrow$  non-trivial solution

There unique solution to  $\bar{X} [sI - A] = 0$  if

$$\det |sI - A| = 0$$

characteristic equation.

The characteristic equation is a polynomial in  $s$  and the roots of the polynomial can be solved for.

Once we have  $s$  (the roots) then we can solve for  $\bar{X}$ .

Example of a  $2 \times 2$   $A$  matrix:  $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$\det \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \right| = 0$$

$$\det \begin{vmatrix} s-a_{11} & -a_{12} \\ -a_{21} & s-a_{22} \end{vmatrix} = 0$$

$$(s-a_{11})(s-a_{22}) - (-a_{21})(-a_{12}) = 0$$

⇓

$$s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21} = 0$$