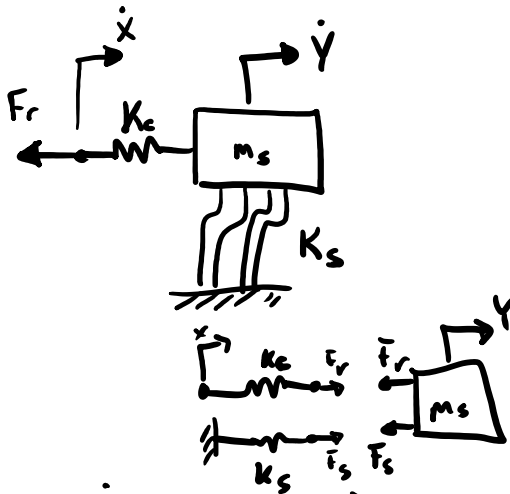


Lab 4 Tips

Given equations:

$$M \frac{d^2 X}{dt^2} + C \frac{dX}{dt} + K X = k_2 i + F_r = k_2 i + K_c (Y - X)$$

$$R i + L \frac{di}{dt} = e_{in} - k_1 \frac{dX}{dt}$$



$$m_s \frac{d^2 Y}{dt^2} = -F_s - F_r = -K_s Y - K_c (Y - X)$$

$$v_x = \dot{X}$$

$$v_y = \dot{Y}$$

$$(1) \quad M \dot{v}_x = -C v_x - K X + k_2 i + K_c (Y - X)$$

$$(2) \quad m_s \dot{v}_y = -K_s Y - K_c (Y - X)$$

$$(3) \quad L \frac{di}{dt} = e_{in} - k_1 v_x - R i$$

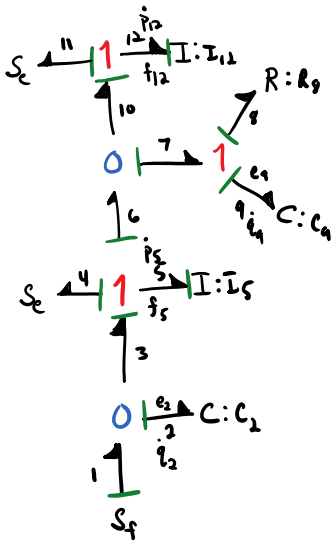
$$(4) \quad \dot{X} = v_x \quad \dot{q}_x = \frac{P_x}{M}$$

$$(5) \quad \dot{Y} = v_y \quad \dot{q}_y = \frac{P_y}{m_s}$$

$$(6) \quad (X - Y) = v_x - v_y \quad \dot{q}_{yx} = \frac{P_x}{M} - \frac{P_y}{m_s}$$

$$\textcircled{1} \dot{q}_2 = f_2 = f_1 - f_3 = S_{f_1} - f_3$$

$$\dot{q}_2 = S_{f_1} - \frac{P_5}{I_5}$$



$$\textcircled{2} \dot{p}_5 = e_5 = e_3 - e_4 - e_6 = e_2 - S_{e_4} - e_7$$

$$\dot{p}_5 = \frac{q_2}{c_2} - S_{e_4} - (e_8 + e_9)$$

$$\dot{p}_5 = \frac{q_2}{c_2} - S_{e_4} - (R_8 f_8 + \frac{q_9}{c_9})$$

$$\dot{p}_5 = \frac{q_2}{c_2} - S_{e_4} - (R_8 [f_6 - f_{10}] + \frac{q_9}{c_9})$$

$$\dot{p}_5 = \frac{q_2}{c_2} - S_{e_4} - (R_8 [\frac{P_5}{I_5} - \frac{P_{12}}{I_{12}}] + \frac{q_9}{c_9})$$

$$\textcircled{3} \dot{q}_9 = f_9 = f_7 = f_6 - f_{10} = \frac{P_5}{I_5} - \frac{P_{12}}{I_{12}}$$

$$\textcircled{4} \dot{p}_{12} = e_{12} = e_{10} - e_{11} = e_7 - S_{e_{11}} = e_8 + e_9 - S_{e_{11}}$$

$$\dot{p}_{12} = R_8 f_8 + \frac{q_9}{c_9} - S_{e_{11}} = \underbrace{R_8 \left(\frac{P_5}{I_5} - \frac{P_{12}}{I_{12}} \right) + \frac{q_9}{c_9} - S_{e_{11}}}_{4 \times 3}$$

$$\dot{\bar{x}} = A \bar{x} + B \bar{u}$$

$$\bar{x} = \begin{bmatrix} q_2 \\ p_5 \\ q_9 \\ p_{12} \end{bmatrix}$$

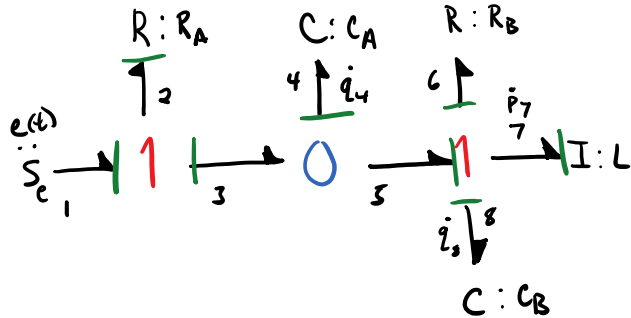
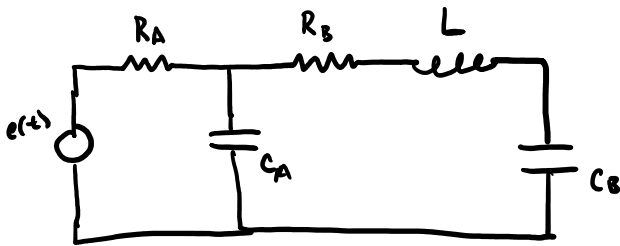
$$\bar{u} = \begin{bmatrix} S_{f_1} \\ S_{e_4} \\ S_{e_{11}} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -\frac{1}{I_5} & 0 & 0 \\ \frac{1}{c_2} & -\frac{R_8}{I_5} & -\frac{1}{c_9} & \frac{R_8}{I_{12}} \\ 0 & \frac{1}{I_5} & 0 & -\frac{1}{I_{12}} \\ 0 & \frac{R_8}{I_5} & \frac{1}{c_9} & -\frac{R_8}{I_{12}} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Exercise

$$f_2 = \frac{e_2}{R_2}$$



States: q_4, p_7, q_8

Inputs: s_e

$$\dot{q}_4 = \frac{1}{R_A} \left(e(t) - \frac{q_4}{C_A} \right) - \frac{p_7}{L}$$

replace with constants after

$$\dot{q}_4 = f_4 = f_3 - f_5 = f_2 - f_7 = \frac{e_2}{R_2} - \frac{p_7}{I_7} = \frac{1}{R_2} (s_{e1} - e_3) - \frac{p_7}{I_7}$$

$$\textcircled{1} \quad \dot{q}_4 = \frac{1}{R_2} \left(s_{e1} - \frac{q_4}{C_4} \right) - \frac{p_7}{I_7}$$

$$\textcircled{2} \quad \dot{p}_7 = e_7 = e_5 - e_6 - e_8 = \frac{q_4}{C_4} - R_6 f_6 - \frac{q_8}{C_8} = \left[\frac{q_4}{C_4} - R_6 \left(\frac{p_7}{I_7} \right) - \frac{q_8}{C_8} \right]$$

$$\textcircled{3} \quad \dot{q}_8 = f_8 = f_7 = \left[\frac{p_7}{I_7} \right]$$

Output equations of state space form:

$$\bar{y} = C \bar{x} + D \bar{u}$$

\uparrow output vector \uparrow output matrix \uparrow feed forward matrix

$q_4 \quad p_7 \quad q_8$

s_{e1}

$$\bar{y} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \end{bmatrix} = \begin{bmatrix} s_{e1} \\ s_{e1} - \frac{q_4}{c_4} \\ q_4/c_4 \\ q_4/c_4 \\ q_4/c_4 \\ R_L \frac{P_7}{I_7} \\ q_4/c_4 - R_L \frac{P_7}{I_7} - \frac{q_8}{c_8} \\ q_4/c_8 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{1}{c_4} & 0 & 0 \\ \frac{1}{c_4} & 0 & 0 \\ \frac{1}{c_4} & 0 & 0 \\ \frac{1}{c_4} & 0 & 0 \\ 0 & \frac{R_L}{I_7} & 0 \\ \frac{1}{c_4} & -\frac{R_L}{I_7} & -\frac{1}{c_8} \\ 0 & 0 & \frac{1}{c_8} \end{bmatrix} \quad D = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{\bar{x}} = A\bar{x} + B\bar{u} \quad \text{state equation} \quad \text{only if linear system!}$$

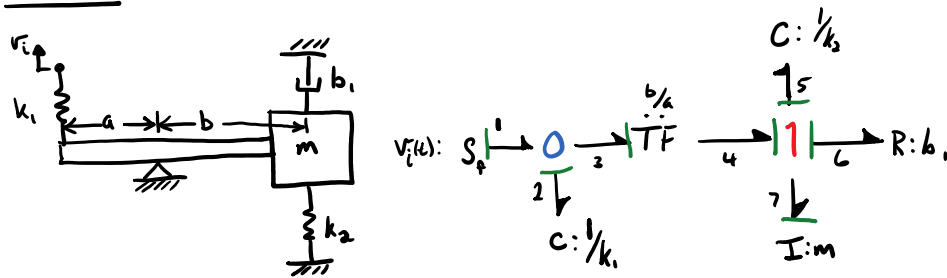
$$\bar{y} = C\bar{x} + D\bar{u} \quad \text{output equation}$$

$$\bar{y} = [P_i] = [c_i f_i] = \left[s_{e1} \frac{1}{R_2} \left(s_{e1} - \frac{q_4}{c_4} \right) \right]$$

$$\bar{y} = [P_i] = \left[s_{e1}^2 / R_2 - \frac{s_{e1} q_4}{R_2 c_4} \right] \quad \text{nonlinear!}$$

$$\bar{y} = f(s_{e1}, q_4)$$

Example



States: q_2, q_5, P_7

Inputs: S_{f1}

$$e_3 = \frac{b}{a} e_4$$

$$\frac{b}{a} f_3 = f_4$$

$$\dot{q}_2 = f_2 = f_1 - f_3 = S_{f1} - \frac{a}{b} f_4 = S_{f1} - \frac{a}{b} f_7 = S_{f1} - \frac{a}{b} \frac{P_7}{I_7}$$

$$\textcircled{1} \quad \dot{q}_2 = v_i(t) - \frac{a}{bm} P_7$$

$$\dot{q}_5 = f_5 = f_7 = \frac{P_7}{I_7}$$

$$\textcircled{2} \quad \dot{q}_5 = \frac{1}{m} P_7$$

$$\dot{P}_7 = e_7 = e_4 - e_5 - e_6 = \frac{a}{b} e_3 - \frac{q_5}{c_5} - R_6 f_6 = \frac{a}{b} e_2 - \frac{q_5}{c_5} - R_6 f_7$$

$$\dot{P}_7 = \frac{a}{b} \frac{q_2}{c_2} - \frac{q_5}{c_5} - R_6 \frac{P_7}{I_7}$$

$$\textcircled{3} \quad \dot{P}_7 = \frac{k_1 a}{b} q_2 - k_2 q_5 - \frac{b_1}{m} P_7$$

State Space

$$\bar{x} = \begin{bmatrix} q_2 \\ q_5 \\ p_7 \end{bmatrix}$$

$$\bar{u} = [v_i(t)]$$

$$A = \begin{bmatrix} 0 & 0 & -\frac{1}{b} \frac{a}{z} \\ 0 & 0 & \frac{3}{z} - \frac{1}{b} \frac{a}{z} \\ \frac{k_1 a}{b} & -k_2 & \frac{1}{z} \frac{a}{z} \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$