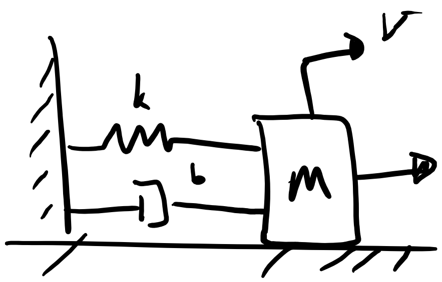
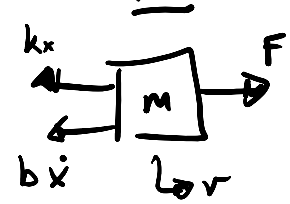
Models of physicals are describes in Various forms, for example:



FBD

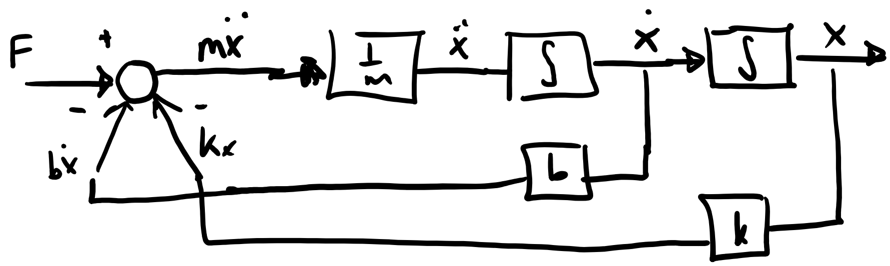


$$\Sigma F = \underbrace{F - kx - b\dot{x}}\ = m\ddot{x}$$

$$m\ddot{x} + b\dot{x} + kx = F$$

2nd ordinary differential equation

Block Diagram



Transfer Function

express a linear ODE in "Laplace domain" to get an algebraic equation.

$$m\ddot{x} + b\dot{x} + kx = F$$

$\Downarrow$ laplace transformation

$$(ms^2 + bs + K)X(s) = F(s)$$

$$\underbrace{\frac{X}{F} \quad \frac{1}{ms^2+bs+k}}_{\text{transfer function}}$$

$$F \longrightarrow \boxed{\frac{1}{ms^2+bs+k}} \longrightarrow X$$

n^{th} order ODE can be converted to n 1^{st} order differential equations.

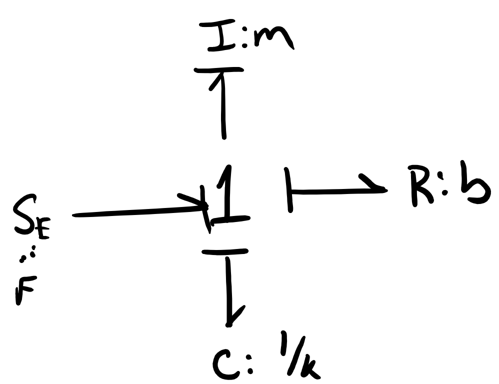
$$\boxed{\begin{aligned} &① \ v = \dot{x}, \quad \dot{v} = \ddot{x} \\ &② \ m\dot{v} + bv + kx = F \end{aligned}} \xrightarrow{\text{explicit form}} \begin{aligned} \dot{x} &= v \\ \dot{v} &= \frac{-bv-kx+F}{m} \end{aligned}$$

State Space form (Matrix Form)

$$\underset{2\times1}{\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix}} = \underset{2\times2}{\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} x \\ v \end{bmatrix}} + \underset{2\times1}{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}} \underset{1\times1}{F}$$

$$\dot{\bar{q}} = A\bar{q} + B\bar{u}$$

New form: Bond graph



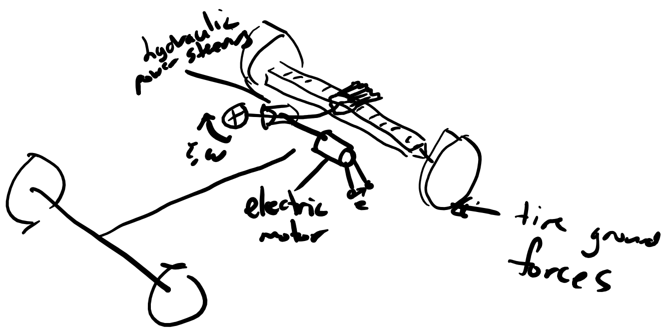
Why is a bond graph useful?

#1 allows modeler to describe a complex system made up of many domains (electrical, hydraulic, magnetic, translational, rotational, etc) in a single common graphical language

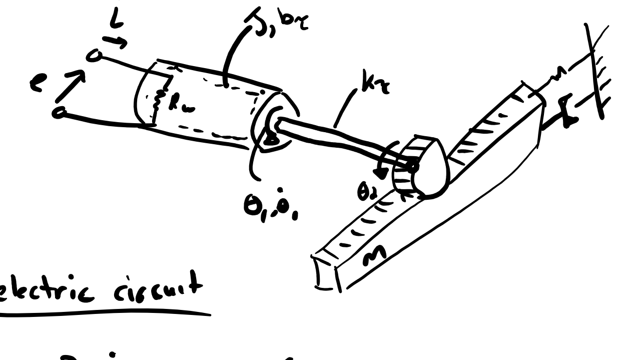
#2 automatic generation of the dynamic equations in State Space form

#3 non-linear and linear systems easily handled

Systems like a car steering system:



Similar, simpler system



electric circuit

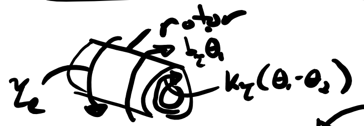
① $e - R_w i - e_{mf} = 0$

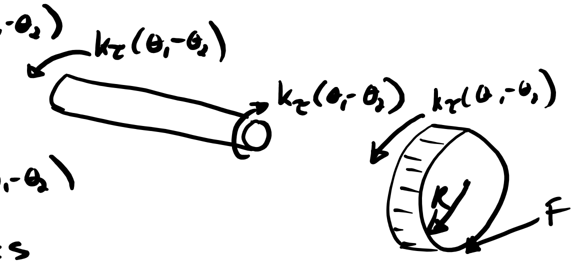
applied voltage; voltage drop; back electromotive force

.

Rotational dynamics

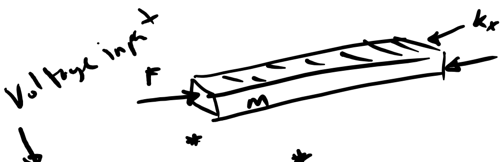
FBDs





② $J\ddot\theta_1 = \tau_e - b_\tau\dot\theta_1 - k_\tau(\theta_1-\theta_2)$

Linear dynamics



③ $F - b\dot x - kx = m\ddot x$

$$\underline{e} - R_w \underline{i} - \underline{e_{mf}} = 0$$
$$J\underline{\ddot\theta_1} = \underline{\tau_e} - b_\tau \underline{\dot\theta_1} - k_\tau(\underline{\theta_1} - \underline{\theta_2})$$
$$\underline{F} - b\underline{\dot x} - k\underline{x} = m\underline{\ddot x}$$
$$k_\tau(\underline{\theta_1} - \underline{\theta_2}) = \underline{F}R$$

7 unknowns: $x, \theta_1, \theta_2, i, \tau_e, e_{mf}, F$

4 eqs

underlined: functions of time

not underlined: system parameters/constants

$$\underline{\tau_e} = T\underline{i}$$
$$\underline{e_{mf}} = T\underline{\dot\theta_1}$$
$$R\underline{\dot\theta_2} = \underline{\dot x}$$

3 more equations

Laplace form

$$e - R_w I - e_{mf} = 0$$
$$J\theta_1 s^2 = \tau_e - b_\tau \theta_1 s - k_\tau(\theta_1 - \theta_2)$$
$$F - bXs - KX = mXs^2$$
$$k_\tau(\theta_1-\theta_2) = FR$$
$$\tau_e = TI$$
$$e_{mf} = T\theta_1 s$$
$$R\theta_2 s = Xs$$

X, θ_1, θ_2, I } 4 primary variables

e -> input

What would the bond graph look like?