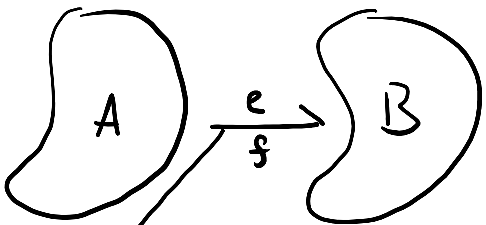
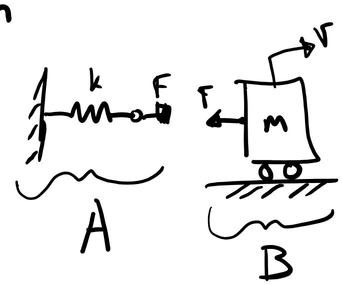
Bond Graph Basics

components A and B of system





Bond represents power flow
$\underbrace{\text{from A to B}}_{\text{corresponds to the arrow}}$

$\underline{P} = Fv$

General terms:
<u>effort</u>
<u>flow</u>

$\underline{P} = e \cdot f$

Power = effort · flow

$E = \int \underline{P}\, dt = \int e \cdot f\, dt$
↑
energy

<u>Sign convention</u>

+ effort X + flow = + power

$A \xrightarrow[+f]{+e} B$
Power goes from A to B

<u>Other important variables</u>

"momentum" $\quad p = \int e\, dt \quad$ or $\quad e = \dfrac{dp}{dt}$

"displacement" $\quad q = \int f\, dt \quad$ or $\quad f = \dfrac{dq}{dt}$

Effort, flow, momentum, displacement for different Domains [all SI units!!!]:

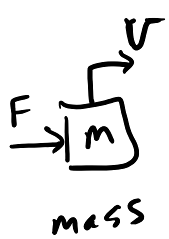
e	f	p	q

	e	f	p	q
translation	F [N] force	v [m/s] velocity	p [N·s] momentum	x [m] displacement
rotation	τ [N·m] torque	ω [rad/s] ang. vel.	h [N·m·s] ang. momentum	θ [rad] angle
electrical	e [V] voltage	i [A] current	λ [V·s] flux	q [A·s] Coulomb
hydraulic	P [N/m^2] pressure	Q [m^3/s] flow rate	P_p [$\frac{N}{m^2}\cdot s$] pressure momentum	V [m^3] volume

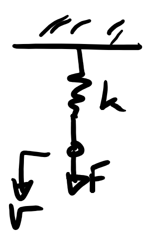
Examples of system components

pwA: bond

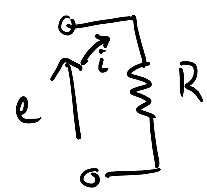
1 port



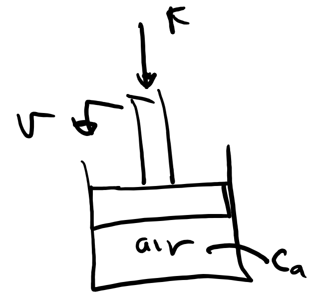
$\frac{F}{v} \rightharpoonup I : m$



$\frac{F}{v} \rightharpoonup C : 1/k$

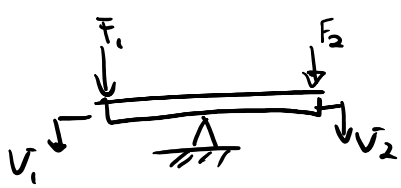


$\frac{e}{i} \rightharpoonup R : R$

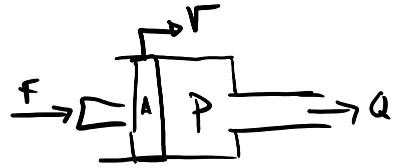


$\frac{F}{v} \rightharpoonup C : C_a$

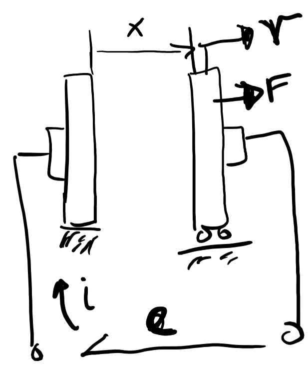
2 port



$\frac{F_1}{v_1} \rightharpoonup TF \frac{F_2}{v_2} \rightharpoonup$

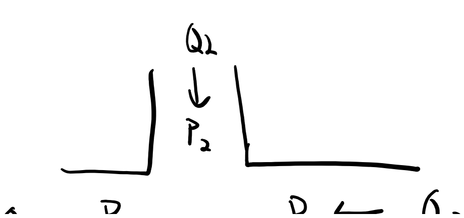


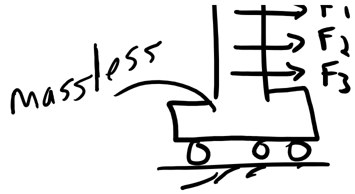
$\frac{F}{v} \rightharpoonup TF \frac{P}{Q} \rightharpoonup$

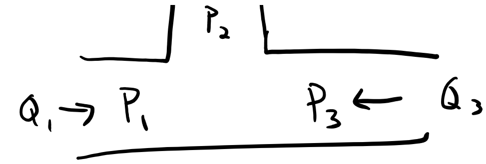


$\frac{e}{i} \rightharpoonup TF \frac{F}{v} \rightharpoonup$

3 port







$v_2 \downarrow F_2$

$\xrightarrow[v_1]{F_1} 1 \xleftarrow[v_3]{F_3}$

$v_1 = v_2 = v_3$

common flow

$F_1 + F_2 + F_3 = 0$

$\downarrow Q_2$

$\xrightarrow[Q_1]{} 0 \xleftarrow[Q_3]{}$

$P_1 = P_2 = P_3$

common effort

$Q_1 + Q_2 + Q_3 = 0$

$v_1 \downarrow F_1$

$\xrightarrow[v_2]{F_2} 1 \xrightarrow[v_3]{F_3}$ m

$F_1 + F_2 = m \cdot \dot{v}$

$F_1 + F_2 - \underbrace{F_3}_{m\dot{v}} = 0$

Generalized form of basic components

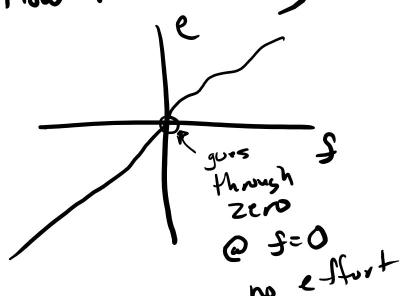
1 port resistance (effort/flow relationship)

$\xrightarrow[f]{e} R$

$e = \Phi_R(f)$

$f = \Phi_R^{-1}(e)$

non-linear form



dissipate energy

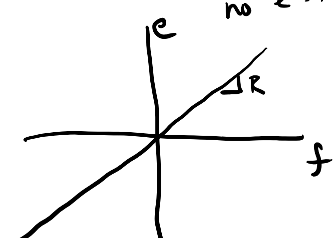
non-conservative

linear form

resistance $e = R f$

$f = \frac{1}{R} e$

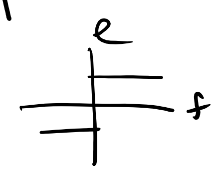
conductance



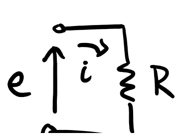
bond symbol

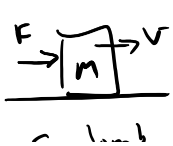
$\xrightarrow[f]{e} R : R$

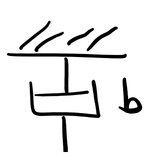
↑ linear constant

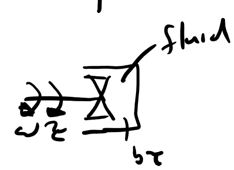


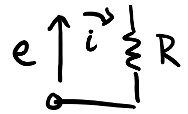
Examples









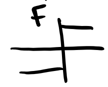


resistor

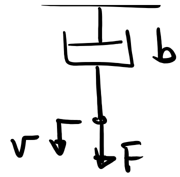
$e = Ri$

$\frac{e}{i} \rightharpoonup R : R$

Coulomb friction

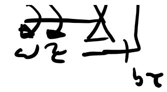


$\frac{F}{v} \rightharpoonup R$



$F = bv$

$\frac{F}{v} \rightharpoonup R : b$



$\tau = b_\tau \omega$

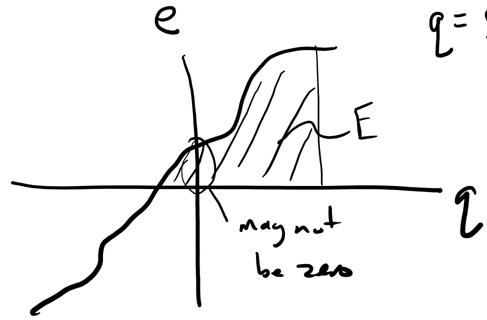
$\frac{\tau}{v} \rightharpoonup R : b_\tau$

1 port compliance (capacitor)

relates effort and displacement and a storage of energy

$\frac{e}{f} \rightharpoonup C$

$q = \Phi_c(e)$

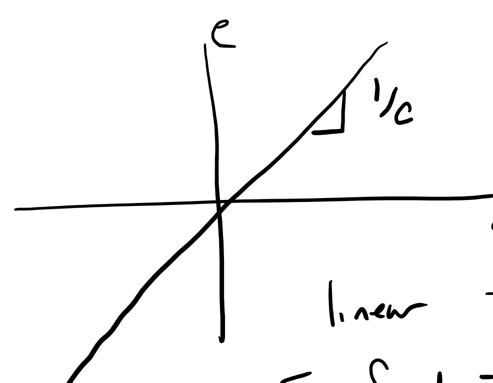


$$E = \int e f\, dt = \int e\, dq$$

$f\,dt = dq$

"potential energy"

energy is conserved



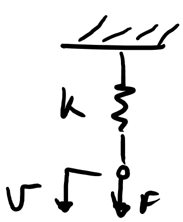
$e = \frac{1}{c} q$

$q = Ce$

linear form

$$E = \int e\, dq = \int \tfrac{1}{c} q\, dq = \frac{q^2}{2C}$$

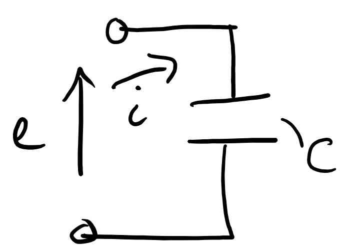
Examples



Spring

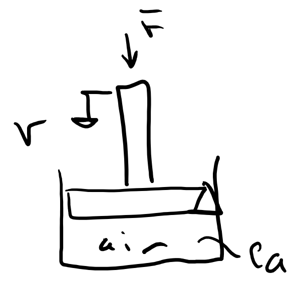
$F = kx$

$x = \int v\, dt$



$q = Ce$

$q = \int i\, dt$



$x = C_a F$

$x = \int v\, dt$

$x = \int v\,dt$ $\qquad$ $q = \int i\,dt$ $\qquad$ $x = \int v$

$\xrightarrow[v]{F} C : 1/k$ $\qquad$ $\xrightarrow[i]{e} C : C$ $\qquad$ $\xrightarrow[v]{F} C : C_a$