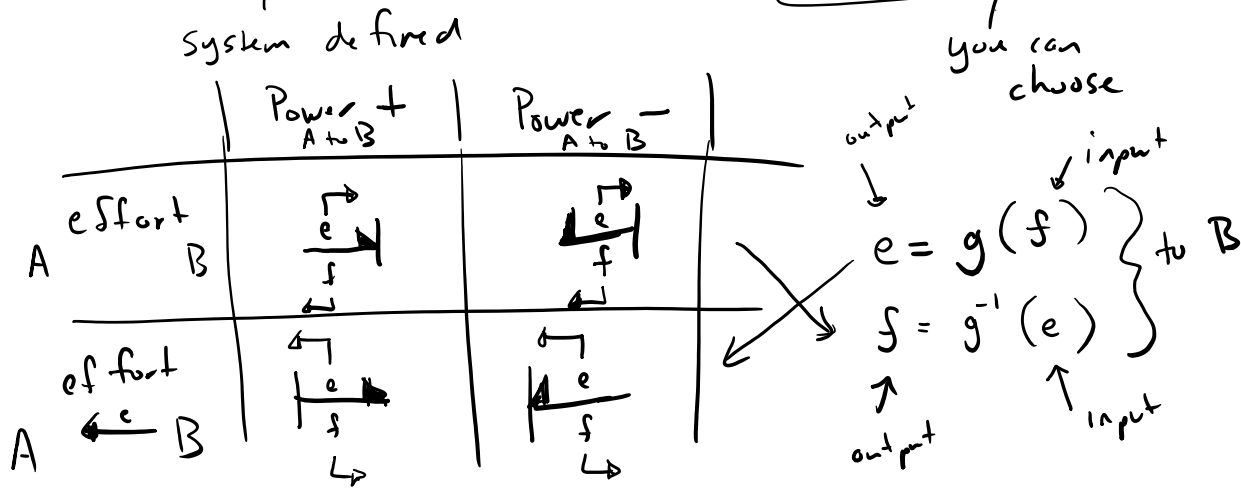
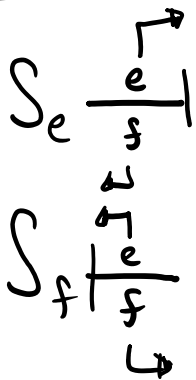


Causality is independent of the power-sign convention.



Causality of Ports

Sources



always have this
Causality by definition

Compliance/Capacitor

causality indicates whether we use the function or its inverse

$$e = \Phi_c^{-1}(q)$$

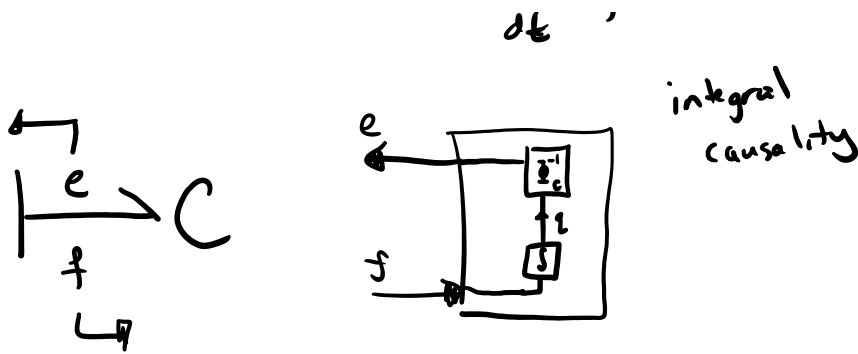
$$q = \Phi_c(e)$$

$$q = \int f dt$$

or

$$\frac{dq}{dt} = f$$

integral

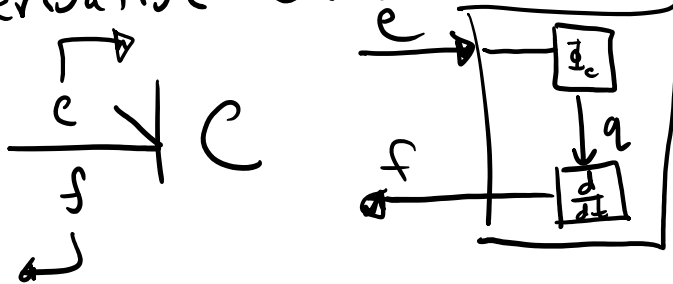


$$e = \Phi_c^{-1}(f dt) \quad \text{non-linear}$$

$$e = \frac{1}{C} \int f dt \quad \text{linear}$$

$$e = \frac{1}{C} q$$

Derivative causality



non-linear

$$f = \frac{d}{dt} (\Phi_c(e))$$

linear

$$f = C \frac{d}{dt} e$$

Similarly for the inertia port

$$f = \Phi_I^{-1}(p)$$

$$p = \Phi_I(f)$$

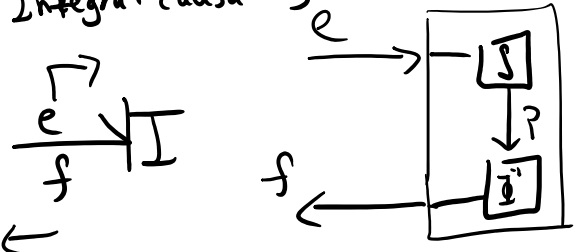
$$\dot{p} = \frac{d}{dt} \Phi_I(f)$$

$$p = \frac{d}{dt} \Phi_I(f)$$

$$\dot{p} = e$$

$$p = \int e dt$$

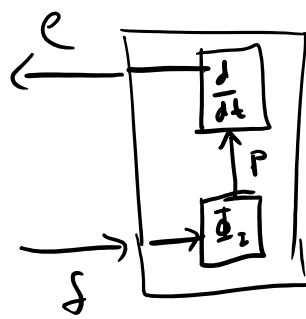
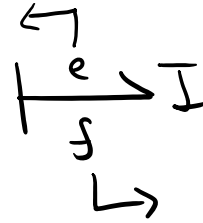
Integral causality



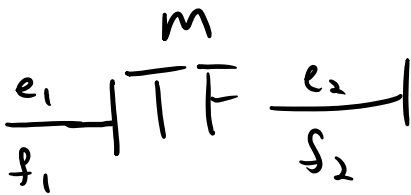
derivative causality

$$e = \frac{d}{dt} \Phi_I(f)$$

derivative causality



Transformer: two possibilities



$$e_2 = \frac{1}{m} e_1$$

↑ input

$$f_1 = \frac{1}{m} f_2$$



$$e_1 = m e_2$$

$$f_2 = m f_1$$

↑ input

Gyrator



$$f_1 = \frac{1}{r} (e_2)_{\text{input}}$$

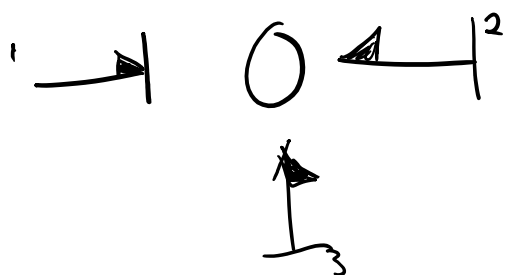
$$f_2 = \frac{1}{r} (e_1)_{\text{input}}$$



$$e_1 = r (f_2)_{\text{input}}$$

$$e_2 = r (f_1)_{\text{input}}$$

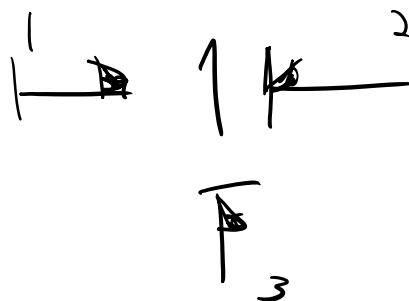
3 ports



$$e_2 = e_3 = e_1 \leftarrow \text{input}$$

$$f_1 = -(f_2 + f_3)$$

↑
output



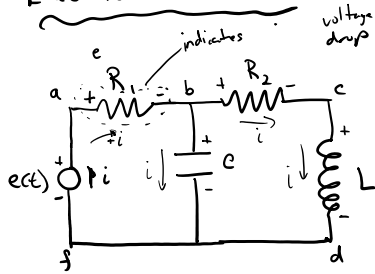
$$f_2 = f_3 = f_1 \leftarrow \text{input}$$

$$e_1 = -(e_2 + e_3)$$

↑
output

System Models

Electrical Networks



Step 1: assign power sign convention

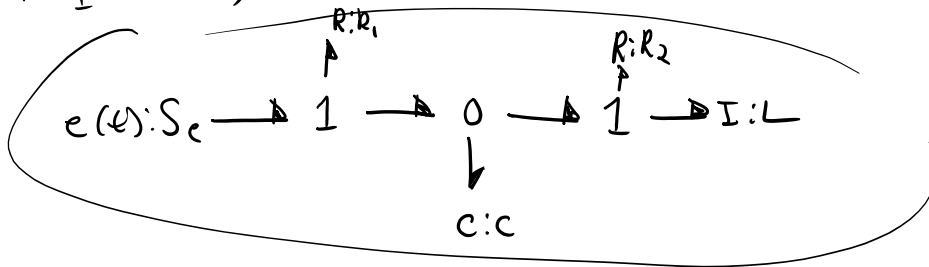
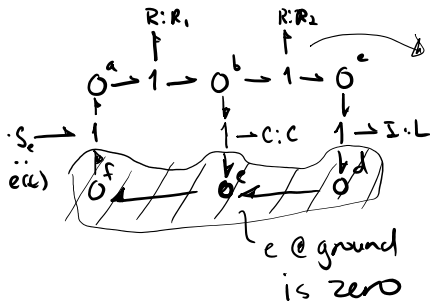
Step 2: Create a 0 junction for each node where there is a different voltage

Establish voltage drops with 1 junctions

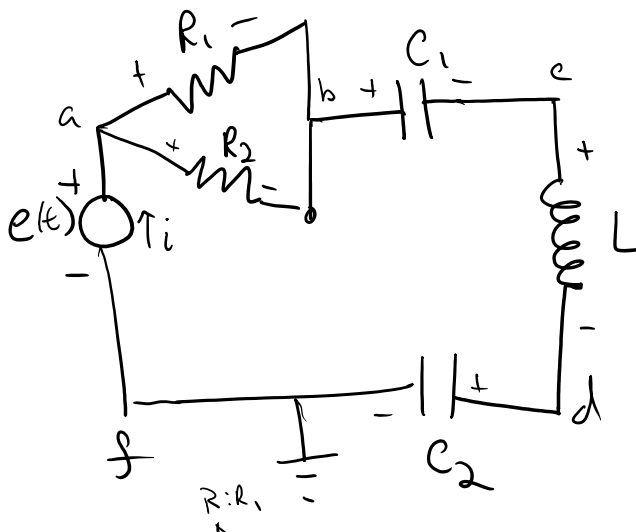
sign convention ensures power flow to ports when power is positive

Step 4: Choose ground or reference voltage node and remove zero power nodes

Step 5: simplify



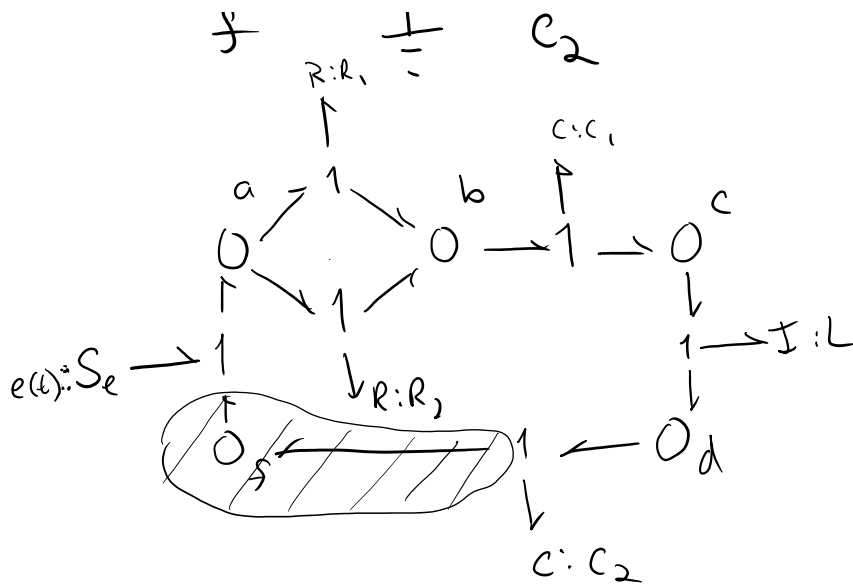
Exercise



Step 1: Assign power sign convention

Step 2: Create 0 junction for each node with common voltage

Step 3: Create 1 junction to establish voltage



to establish voltage drops

Step 4: choose ground/ref voltage, set to zero, eliminate bonds of zero power

Step 5: simplify

