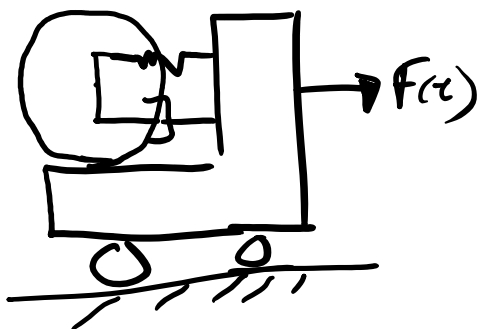
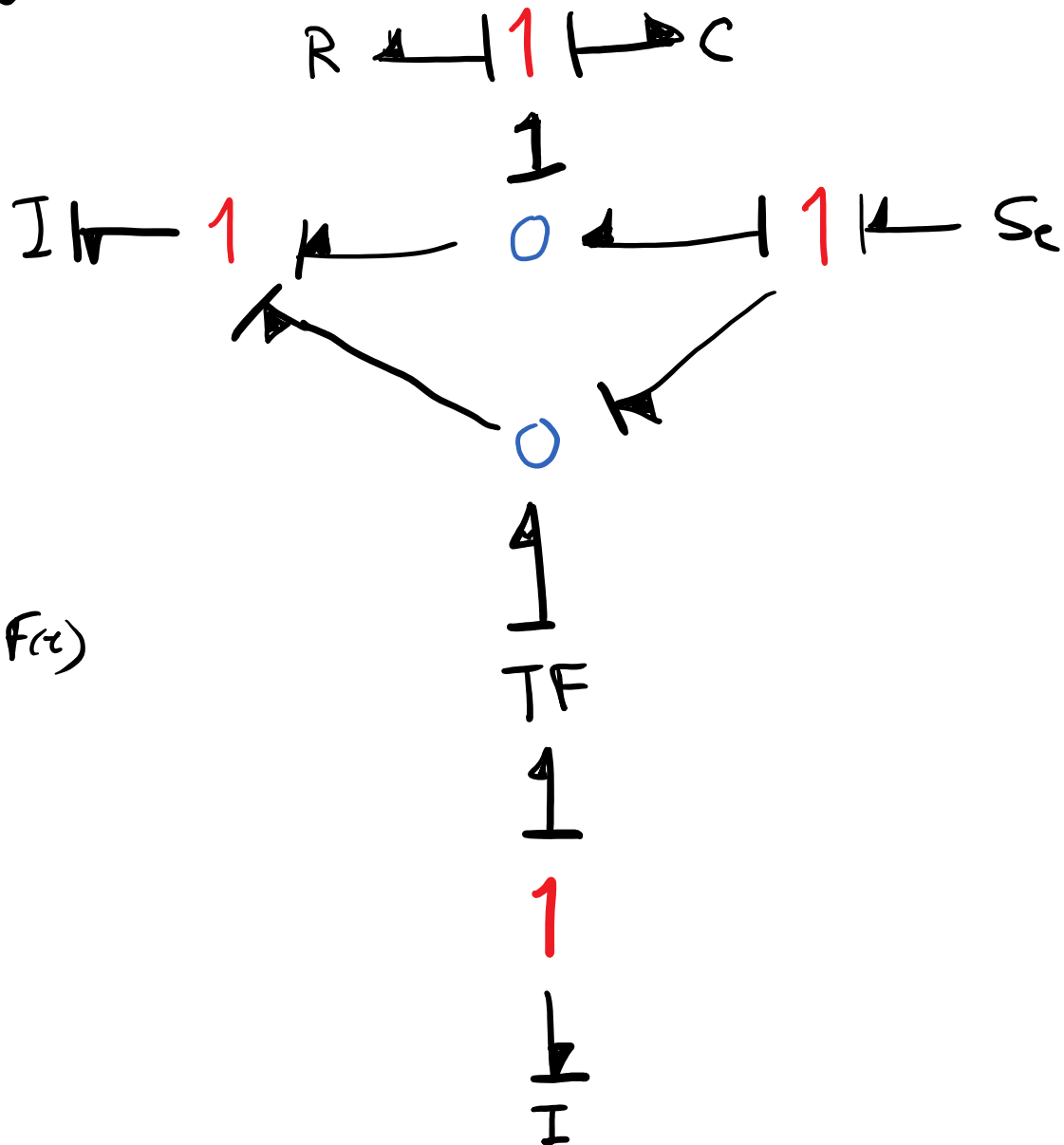


# Assign Causality



$$\frac{e_1}{f_1} \xrightarrow{k_1, k_2} GY \xrightarrow{e_2} \frac{f_2}{f_2}$$

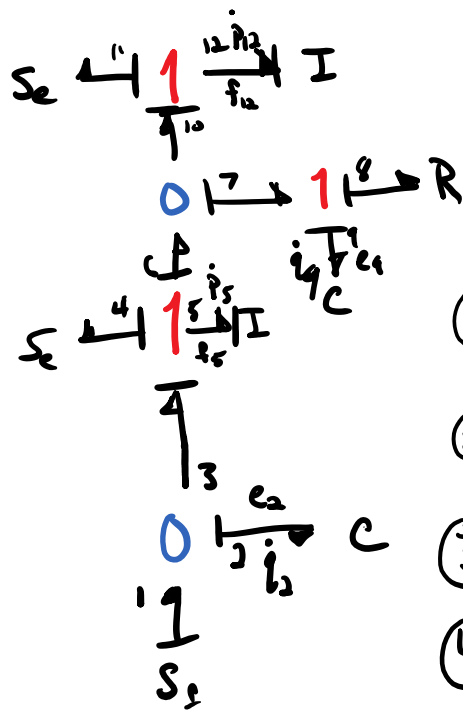
$$e_1 = k_1 f_2$$

$$k_2 f_1 = e_2$$

## Forming System Equations

1. number the bonds on generalized bond graph
2. define input variables: all  $S_f, S_e$
3. define state variables:  $p$ 's and  $q$ 's of all  $I$ 's and  $C$ 's with integral causality
4. write  $\dot{p}, f$  and  $\dot{q}, e$  for each  $I$  and  $C$   
 $\underbrace{\quad}_{\text{co-energy variables}}$
5. start with lowest # bond  $I$  or  $C$  and write  $\dot{p} = e$  or  $\dot{q} = f$
6. follow causality for  $e$  or  $f$  to other  $I, C, R$ 's
7. write constitutive relationships for non-state variables

# Quarter Car Model



Inputs:  $S_{f1}, S_{e4}, S_{e11}$

state variables:  $q_2, p_5, q_9, p_{12}$

only from  
I, C's with integral  
causality

$$\textcircled{1} \dot{q}_2 = f_2$$

$$\textcircled{2} \dot{p}_5 = e_5$$

$$\textcircled{3} \dot{q}_4 = f_9$$

$$\textcircled{4} \dot{p}_{12} = e_{12}$$

$$\textcircled{1} \dot{q}_2 = f_2 = f_1 - f_3 = S_{f1} - f_5 = S_{f1} - \frac{p_5}{I_5}$$

$\uparrow$  caused by  $f_1, p_3$        $\uparrow$  caused by  $f_5$        $\uparrow$   $f_5$  is related to  $p_5$

$$\textcircled{1} \dot{q}_2 = S_{f1} - \frac{p_5}{I_5}$$

$$\textcircled{2} \dot{p}_5 = e_5 = e_3 - e_4 - e_6 = e_2 - e_4 - e_6$$

$\uparrow$  caused by  $e_3, e_4, e_6$        $\uparrow$  caused by  $e_2$        $\uparrow$   $e = \frac{p}{c}$        $\uparrow$  caused by  $e_7$

$$\dot{p}_5 = \dot{q}_2 \quad n \quad ( \dots + 0 )$$

$$15 = \frac{q_2}{c_2} - S_{e4} - (e_8 + e_9)$$

$$\dot{P}_5 = \frac{q_2}{c_2} - S_{e4} - \left( R_8 f_8 + \frac{q_1}{c_1} \right)$$

$$f_1 = f_7 = f_8 = f_6 - f_{10} = \frac{P_5}{I_5} - \frac{P_{12}}{I_{12}}$$

$$(2) \dot{P}_5 = \frac{q_2}{c_2} - S_{e4} - \left[ R_8 \left( \frac{P_5}{I_5} - \frac{P_{12}}{I_{12}} \right) + \frac{q_1}{c_1} \right]$$

$$(3) \dot{q}_1 = f_9 = \frac{P_5}{I_5} - \frac{P_{12}}{I_{12}}$$

$$(4) \dot{P}_{12} = e_{12} = e_{10} - e_{11} = e_{10} - S_{e11}$$

$$\dot{P}_{12} = (e_8 + e_9) - S_{e11}$$

$$\dot{P}_{12} = \left( R_8 f_8 + \frac{q_1}{c_1} \right) - S_{e11}$$

$$(4) \dot{P}_{12} = \left[ R_8 \left( \frac{P_5}{I_5} - \frac{P_{12}}{I_{12}} \right) + \frac{q_1}{c_1} \right] - S_{e11}$$

State Space Form

$$\dot{\bar{x}} = A\bar{x} + B\bar{u}$$

$$\begin{matrix} 4 \times 1 & 4 \times 4 & 4 \times 1 & 4 \times 3 \\ \begin{bmatrix} \dot{q}_2 \\ \dot{P}_5 \end{bmatrix} & \begin{bmatrix} 0 & -\frac{1}{I_5} & 0 & 0 \\ \frac{1}{c_1} & -\frac{R_8}{I_5} & -\frac{1}{c_1} & \frac{R_8}{I_{12}} \end{bmatrix} & \begin{bmatrix} q_2 \\ P_5 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{matrix} 3 \times 1 \\ S_{f1} \end{matrix} \end{matrix}$$

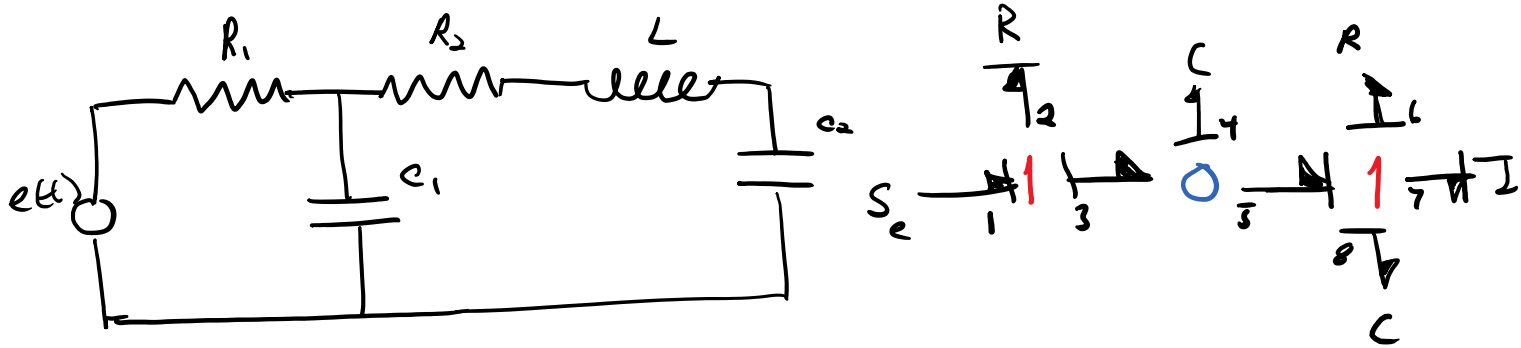
$$\begin{bmatrix} \dot{p}_5 \\ \dot{q}_9 \\ \dot{p}_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{c_2} & -\frac{R_8}{I_5} & -\frac{1}{c_9} & \frac{R_8}{I_{12}} \\ 0 & \frac{1}{I_5} & 0 & -\frac{1}{I_{12}} \\ 0 & \frac{R_8}{I_5} & \frac{1}{c_9} & -\frac{R_8}{I_{12}} \end{bmatrix} \begin{bmatrix} p_5 \\ q_9 \\ p_{12} \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} S_{f1} \\ S_{e4} \\ S_{e11} \end{bmatrix}$$

$$\dot{q}_2 = S_{f1} - \frac{p_5}{I_5}$$

$$\dot{p}_5 = \frac{q_2}{c_2} - S_{e4} - \left[ \frac{q_9}{c_9} + R_8 \left( \frac{p_5}{I_5} - \frac{p_{12}}{I_{12}} \right) \right]$$

$$\dot{q}_9 = \frac{p_5}{I_5} - \frac{p_{12}}{I_{12}}$$

$$\dot{p}_{12} = \left[ R_8 \left( \frac{p_5}{I_5} - \frac{p_{12}}{I_{12}} \right) + \frac{q_9}{c_9} \right] - S_{e11}$$



Input variables:

State variables:

complete before  
Thursday