scalar equation

$$x(t) = \bar{X} e^{st} = \bar{X} e^{(\lambda \pm \omega i)t}$$

$$x(t) = \bar{Y} e^{\lambda t} \sin(\omega t + \phi)$$

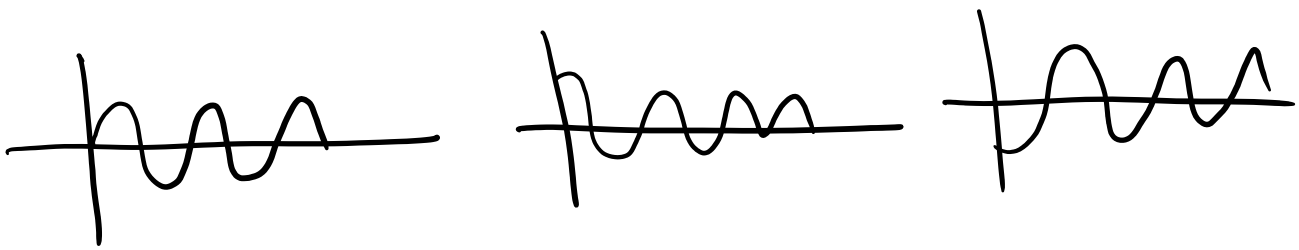
ϕ → phase angle

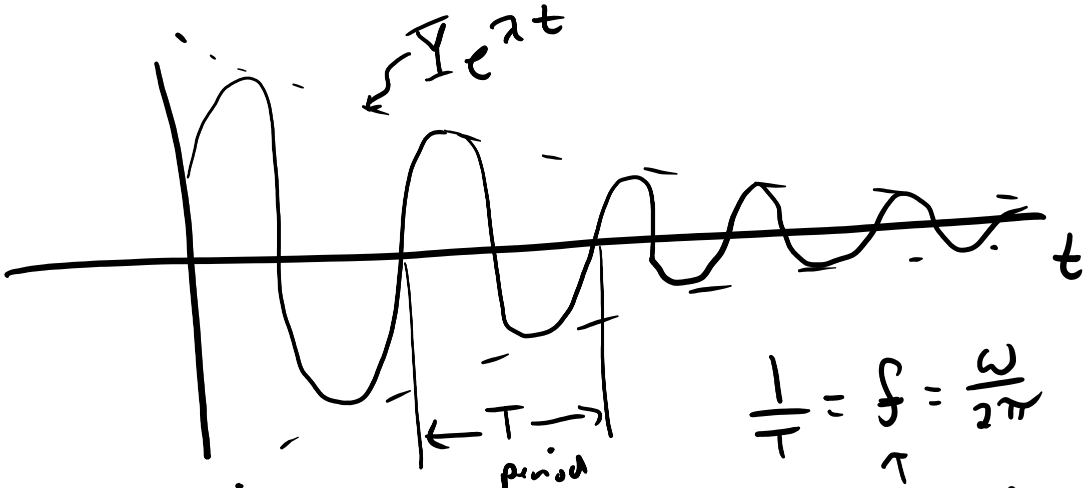
ω governs the frequency of sinusoidal oscillation

λ governs the growth or decay

$$s_{1,2} = \lambda \pm \omega i$$

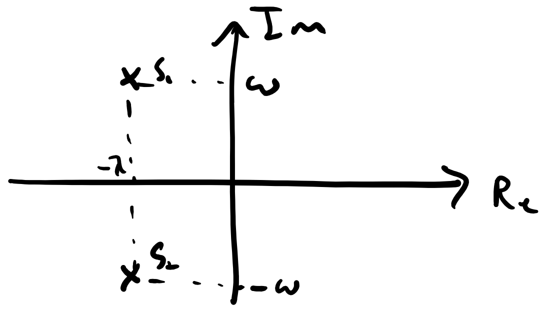
$\bar{Y}, \phi \Rightarrow$ introduce initial conditions: $x_0, \dot{x}_0$





$$\frac{1}{T} = f = \frac{\omega}{2\pi}$$

↑ frequency



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Tuesday, March 5, 2019 11:49 AM

$\Psi \sin(\omega t + \phi)$

$X(t) = \Psi e^{\lambda t}$ $\lambda > 0$

$\Psi e^{\lambda t}$

$\lambda < 0$

$\Psi \sin(\omega t + \phi)$

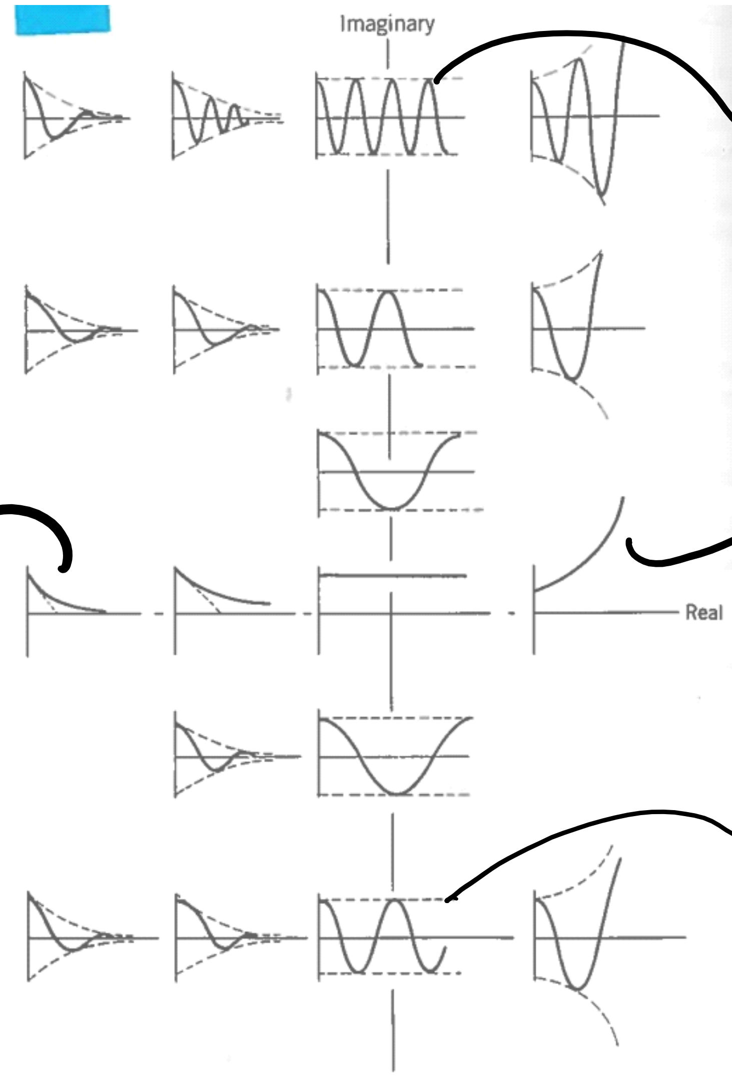


FIGURE 6.8. Correlation of time response with complex eigenvalues.

Eigenvector

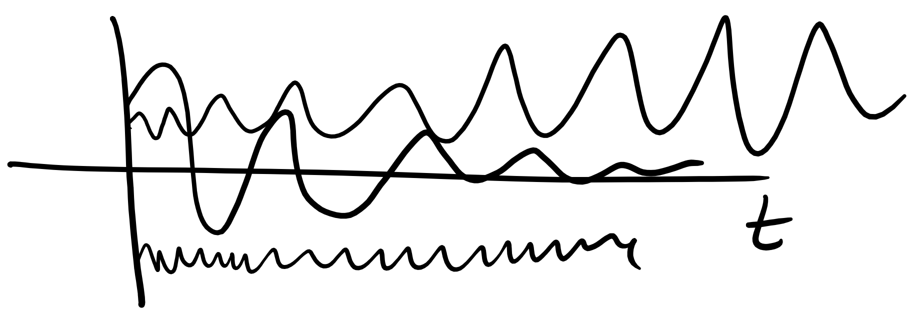
once we know s then plug it back into:

$$\bar{X}[sI-A]=0$$

every eigenvalue has an associated eigenvector

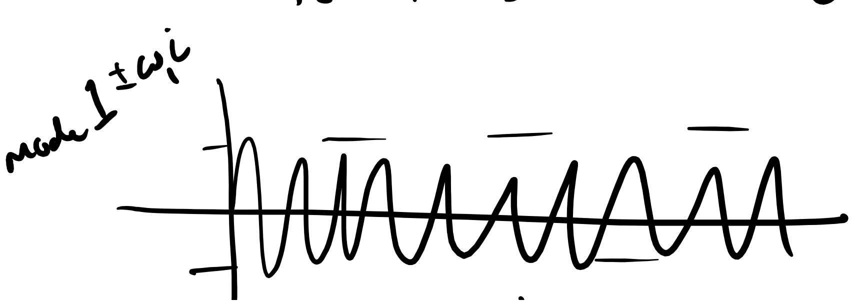
eigenvalues: dictate growth/decay and frequency

eigenvectors: dictate the relative magnitudes and phase among the states

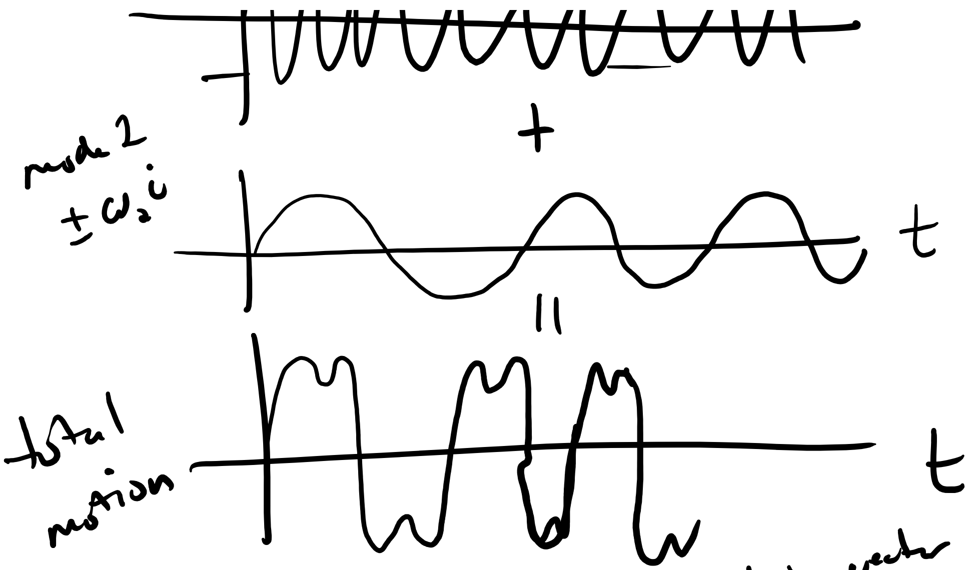


eigenvalue & eigenvector => eigen mode
"mode of motion"

superposition of all eigenmodes gives total state trajectory!!



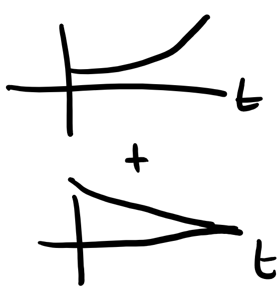
4th order

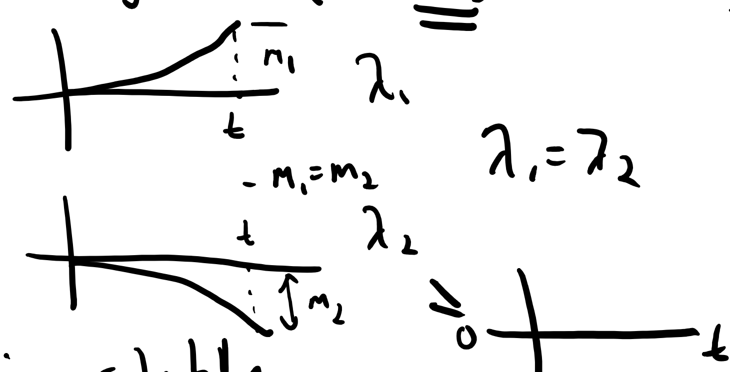


$$\bar{X}(t) = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \Phi_{11} e^{s_1 t} + \overbrace{\Phi_{12}}^{2^{nd}\text{ eigenvector}} e^{s_2 t} + \cdots + \overbrace{\Phi_{1n}}^{n^{th}\text{ eigenvector}} e^{s_n t} \\ \Phi_{21} e^{s_1 t} + \Phi_{22} e^{s_2 t} + \cdots + \Phi_{2n} e^{s_n t} \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ \underbrace{\Phi_{n1} e^{s_1 t}}_{\text{mode } 1} + \underbrace{\Phi_{n2} e^{s_2 t}}_{\text{mode } 2} + \cdots + \underbrace{\Phi_{nn} e^{s_n t}}_{\text{mode } n} \end{bmatrix}$$

n states

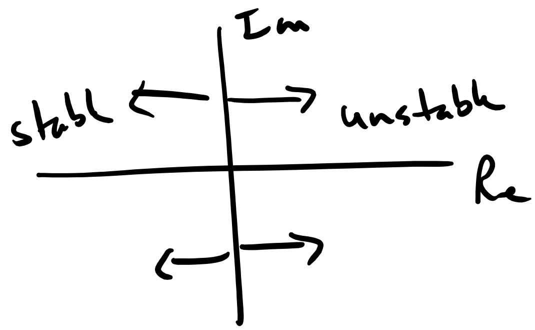
If $\lambda > 0 \Rightarrow$ unstable system (if __any__ λ is > 0)





If all $\lambda < 0 \Rightarrow$ system is stable

If a $\lambda = 0 \Rightarrow$ that mode is "marginally stable"



right half plane
eigenvalues $\Rightarrow$ unstable

left half plane
eigenvalues $\Rightarrow$ stable

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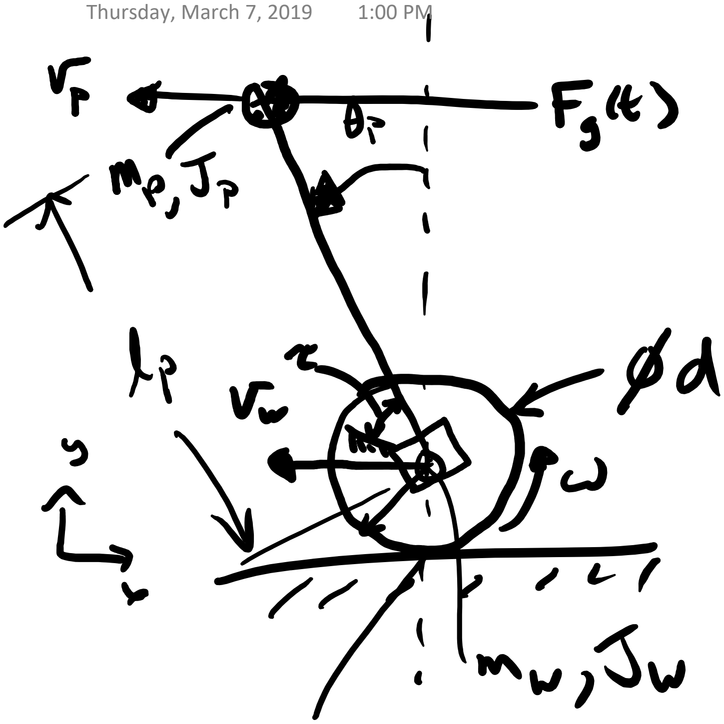
Thursday, March 7, 2019 1:00 PM



θ_p is small angle