

Figure 1.3.1

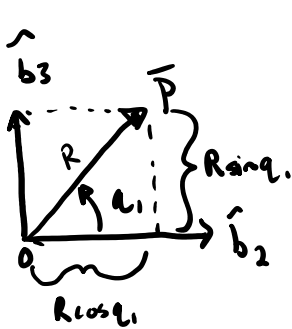
Example: Gimbal, Vectors composed of components in different reference frames

It is possible to construct vectors with different measure numbers associated with different reference frames. These measure numbers can be functions of different scalars in different R.F.s.

$$\bar{p} = \alpha_1 \hat{a}_1 + \alpha_2 \hat{a}_2 + \alpha_3 \hat{a}_3$$

$$\bar{p} = \beta_1 \hat{b}_1 + \beta_2 \hat{b}_2 + \beta_3 \hat{b}_3$$

$\bar{p}$  is vector from O to P



$$\bar{p} = 0 \hat{b}_1 + R \cos(q_1) \hat{b}_2 + R \sin(q_1) \hat{b}_3$$

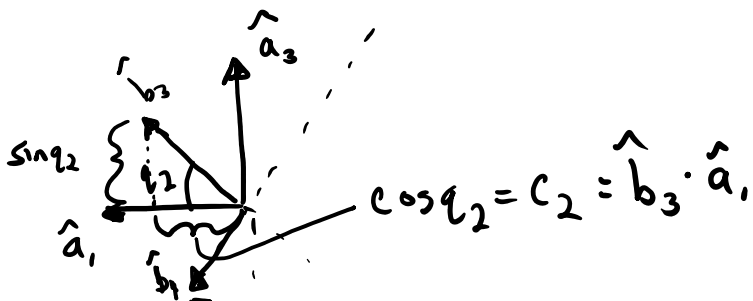
$$\beta_1 = 0 = \bar{p} \cdot \hat{b}_1$$

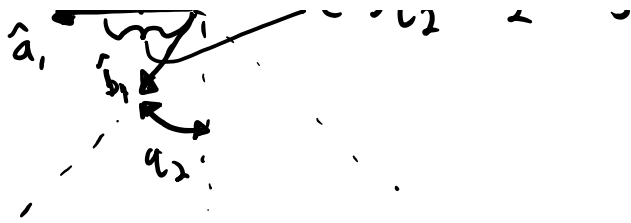
$$\beta_2 = R \cos q_1 = \bar{p} \cdot \hat{b}_2$$

$$\beta_3 = R \sin q_1 = \bar{p} \cdot \hat{b}_3$$

How to calculate  $\alpha_i$ ?

$$\alpha_1 = \bar{p} \cdot \hat{a}_1 = R \cos q_1 \hat{b}_2 \cdot \hat{a}_1 + R \sin q_1 \hat{b}_3 \cdot \hat{a}_1 = R \sin q_1 c_2$$





$$\alpha_2 = \bar{p} \cdot \hat{a}_2 = R_{c_1} \hat{b}_1 \cdot \hat{a}_2 + R_{s_1} \hat{b}_3 \cdot \hat{a}_2$$

$$\alpha_2 = R_{c_1}$$

$$\alpha_3 = \bar{p} \cdot \hat{a}_3 = R_{c_1} \hat{b}_2 \cdot \hat{a}_3 + R_{s_1} \hat{b}_3 \cdot \hat{a}_3$$

$$\alpha_3 = R_{s_1} S_2$$

$$\bar{p} = R_{s_1} c_2 \hat{a}_1 + R_{c_1} \hat{a}_2 + R_{s_1} S_2 \hat{a}_3$$

$\bar{p}$  expressed in both A and B

### Derivatives

Let  $\bar{v}$  be a vector function of  $n$  scalar variables  $q_1, \dots, q_n$  in reference frame A. The  $n$  vectors called first partial derivatives of  $\bar{v}$  in A are denoted by

$${}^A \frac{\partial \bar{v}}{\partial q_r} \quad \text{partial of } \bar{v} \text{ in A wrt } r^{\text{th}} q$$

The partial derivative is defined as:

$${}^A \frac{\partial \bar{v}}{\partial q_r} \triangleq \sum_{i=1}^3 \frac{\partial v_i}{\partial q_r} \hat{a}_i \quad \text{for } r=1, \dots, n$$

$$\bar{v} = v_1 \hat{a}_1 + v_2 \hat{a}_2 + v_3 \hat{a}_3$$

$$= \frac{\partial v_1}{\partial q_r} \hat{a}_1 + \frac{\partial v_2}{\partial q_r} \hat{a}_2 + \frac{\partial v_3}{\partial q_r} \hat{a}_3$$

If  $\bar{v}$  is a function of only one variable (e.g. time)

$$\begin{aligned} \frac{d\vec{v}}{dt} &= \frac{d\vec{v}}{dt} = \frac{dv_1}{dt} \hat{a}_1 + \frac{dv_2}{dt} \hat{a}_2 + \frac{dv_3}{dt} \hat{a}_3 \\ &= \dot{v}_1 \hat{a}_1 + \dot{v}_2 \hat{a}_2 + \dot{v}_3 \hat{a}_3 \end{aligned}$$

If  $\vec{v}$  is not a function of  $q$  in  $A$  then

$$\frac{\partial \vec{v}}{\partial q} = 0$$

Example

$$\vec{p} = R_{s_1 c_2} \hat{a}_1 + R_{c_1} \hat{a}_2 + R_{s_1 s_2} \hat{a}_3 = R \hat{c}_2 = R_{c_1} \hat{b}_2 + R_{s_1} \hat{b}_3$$

$$\frac{\partial \vec{p}}{\partial q_1} = R_{c_2} c_1 \hat{a}_1 - R_{s_1} \hat{a}_2 + R_{s_2} c_1 \hat{a}_3$$

$$\frac{\partial \vec{p}}{\partial q_1} = -R_{s_1} \hat{b}_2 + R_{c_1} \hat{b}_3$$

$$\frac{\partial \vec{p}}{\partial q_1} = 0$$

These derivative can be expressed in any frame.

$$\hat{b}_2 = \hat{a}_2 \quad \hat{b}_3 = c_2 \hat{a}_1 + s_2 \hat{a}_3$$

$$\frac{\partial \vec{p}}{\partial q_1} = -R_{s_1} \hat{a}_2 + R_{c_1} (c_3 \hat{a}_1 + s_2 \hat{a}_3)$$