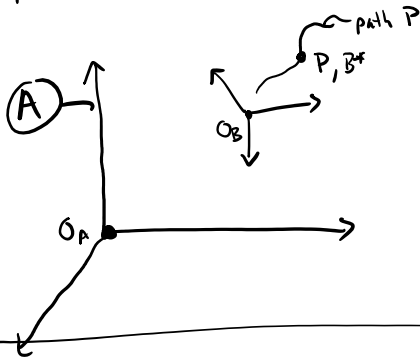


# Midterm

## Chapters 1-2.13

### One point moving on rigid body

One point P moving on/in a rigid body B while B moves in reference frame A. Let B\* be the point fixed in B where P is at this instant of time (ie. that B\* coincides with P presently)



Note:

$$A \vec{\omega}^B \neq 0 \text{ in general}$$

$$A \vec{V}^{O_B} \neq 0 \text{ in general}$$

$$A \vec{V}^P = B \vec{V}^P + A \vec{V}^{B^*}$$

one point velocity theorem

$$A \vec{a}^P = B \vec{a}^P + \underbrace{A \vec{a}^{B^*}}_{\substack{\text{relative} \\ \text{acc}}} + \underbrace{2 \vec{\omega}^B \times B \vec{V}^P}_{\substack{\text{Coriolis} \\ \text{acc}}} + \underbrace{\vec{\omega}^B \times (\vec{\omega}^B \times r)}_{\text{centrifugal acc}}$$

5 acc. terms!  
one point acc. theorem

### Example Fig 2.6.1

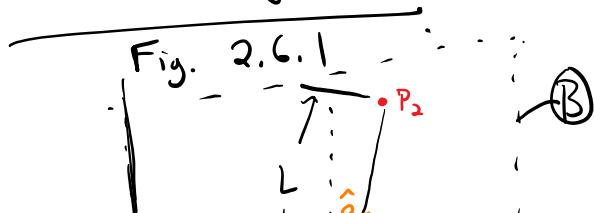
use 1 point theorem

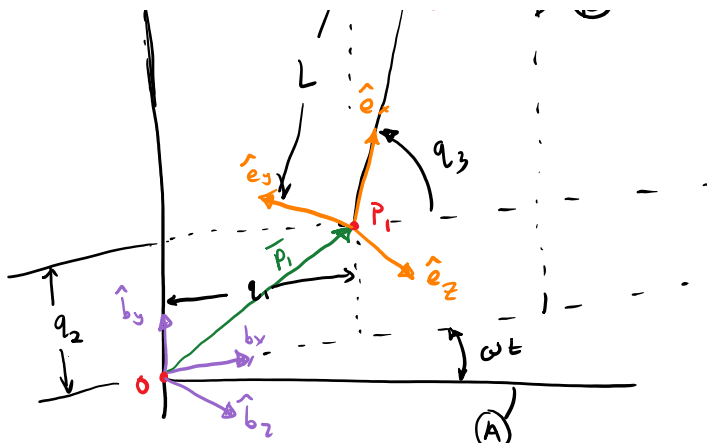
compute  $A \vec{V}^{P_1}$  and  $A \vec{a}^{P_1}$        $O$  and  $B^*$  fixed in B

let  $B^*$  be the point fixed in B where  $P_1$  is now

$$A \vec{V}^{P_1} = B \vec{V}^{P_1} + A \vec{V}^{B^*} = \dot{q}_1 \hat{b}_x + \dot{q}_2 \hat{b}_y - \omega \dot{q}_1 \hat{b}_z$$

### Back to Fig 2.6.1





$${}^A \bar{a} P_i = {}^B \bar{a} P_i + \underbrace{{}^A \bar{a} \cdot \bar{v}^B}_{\dot{q}_1 \hat{b}_x + \dot{q}_2 \hat{b}_y} + \underbrace{2 \bar{a} \cdot \bar{\omega} \times \bar{r}^B}_{-\omega^2 q_1 \hat{b}_x - 2\omega \dot{q}_1 \hat{b}_z}$$

## New Topic. State Variables

State variables of a set  $S$  of  $\nu$  particles  $P_i$  ( $i=1, \dots, \nu$ ) in reference frame  $A$  consist of two parts:

- State of  $S$  in  $A$  is:
- (a) configuration of  $S$  in  $A$  (where they are in  $A$ )
  - (b) motion of  $S$  in  $A$  (how they are moving in  $A$ )

Fundamental question in dynamics: "How does the state change in time?"

Configuration is characterized by position vectors to each particle.

Motion is characterized by velocity vectors of each particle.

When the motion is unconstrained we need  $2\nu$  (→) scalar measures numbers to characterize

vectors or (6D) scalar measures numbers to characterize that state.

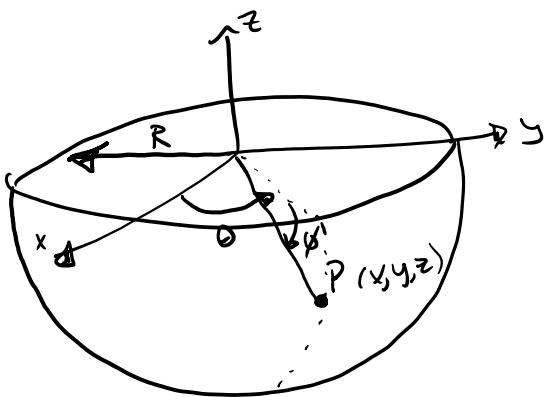
Constraints reduce this number of state variables necessary to fully characterize the state.

Configuration variables are called "generalized coordinates" <sup>GC's</sup> of  $S$  in  $A$ .  
Motion variables are called "generalized speeds" <sup>GS's</sup> of  $S$  in  $A$ .

Both GC's and GS's are functions of time and both can be chosen in an infinite number of ways!

### Example

hemispherical bowl of radius  $R$  with a single particle on its surface



constraint equation  
$$x^2 + y^2 + z^2 = R^2$$

But if we use spherical coordinates to locate  $P$  no constraint equation is needed! Only two coordinates that fully specify the configuration.  $(\theta, \phi)$

Configuration constraints equations are written in this form:

Configuration constraints equations are written in the form.

$$f(x_1, y_1, z_1, \dots, x_n, y_n, z_n, t) = 0$$

$$x^2 + y^2 + z^2 - R^2 = 0$$

this are called holonomic constraints.

- no time derivatives  
of coordinates  
present

- function of only  
position & time

Holonomic constraints subtypes:

a) Rheonomic - when time is explicit

b) Scleronomous - when time is not explicit in  $f$