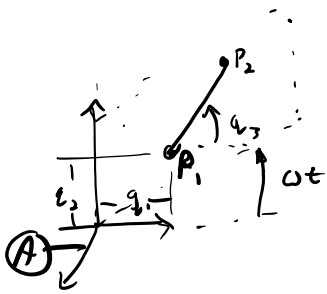


Another partial velocity example

Fig 2.6.1



- ① ${}^A \bar{V}^{P_1} = \dot{q}_1 \hat{b}_x + \dot{q}_2 \hat{b}_y - \omega q_1 \hat{b}_z$
- ② ${}^A \bar{V}^{P_1} = (\dot{q}_1 c_3 + \dot{q}_2 s_3) \hat{e}_x + (-\dot{q}_1 s_3 + \dot{q}_2 c_3) \hat{e}_y - \omega q_1 \hat{e}_z$
- ③ ${}^A \bar{V}^{P_1} = (\dot{q}_1 \cos \omega t - \omega q_1 \sin \omega t) \hat{a}_x + \dot{q}_2 \hat{a}_y - (\dot{q}_1 \sin \omega t + \omega q_1 \cos \omega t) \hat{a}_z$

${}^B \bar{\omega}^E = \dot{q}_3 \hat{e}_z$

anyone one of these expressions can be used to pick 3 generalized speeds

in ①

$u_1 = \dot{q}_1 \quad u_2 = \dot{q}_2 \quad u_3 = \dot{q}_3$

${}^A \bar{V}^{P_1} = u_1 \hat{b}_x + u_2 \hat{b}_y - \omega q_1 \hat{b}_z$

${}^B \bar{\omega}^E = u_3 \hat{e}_z$ no \dot{q} 's!

Partial velocities of P_1 in A

${}^A \bar{V}_1 = \hat{b}_x \quad {}^A \bar{V}_2 = \hat{b}_y \quad {}^A \bar{V}_3 = 0 \quad {}^A \bar{V}_t = -\omega q_1 \hat{b}_z$

${}^B \bar{\omega}_1^E = 0 \quad {}^B \bar{\omega}_2^E = 0 \quad {}^B \bar{\omega}_3^E = \hat{e}_z \quad {}^B \bar{\omega}_t^E = 0$

$\bar{V} = \sum \bar{V}_r u_r + \bar{V}_t$

in ②

$u_1 = \dot{q}_1 c_3 + \dot{q}_2 s_3 \quad u_2 = \dot{q}_2 c_3 - \dot{q}_1 s_3 \quad u_3 = \dot{q}_3$

${}^A \bar{V}^{P_1} = u_1 \hat{e}_x + u_2 \hat{e}_y - \omega q_1 \hat{e}_z$

${}^A \bar{\omega}^E = \omega s_3 \hat{e}_x + \omega c_3 \hat{e}_y + u_3 \hat{e}_z$

${}^A \bar{V}_1 = \hat{e}_x \quad {}^A \bar{V}_2 = \hat{e}_y \quad {}^A \bar{V}_3 = 0 \quad {}^A \bar{V}_t = -\omega q_1 \hat{e}_z$

different u 's give different partial velocities

$${}^A \bar{\omega}_1^E = 0$$

$${}^A \bar{\omega}_2^E = 0$$

$${}^A \bar{\omega}_3^E = \hat{e}_z$$

$${}^A \bar{\omega}_t^E = \omega_{S_3} \hat{e}_x + \omega_{L_3} \hat{e}_y$$

Partial velocities

Every velocity and angular velocity in system S' can be written in terms of the G.S.'s

$$\bar{V} = \sum_{r=1}^n \bar{V}_r u_r + \bar{V}_t$$

$$\bar{\omega} = \sum_{r=1}^n \bar{\omega}_r u_r + \bar{\omega}_t$$

unconstrained/holonomic r^{th} partial (angular) velocities of point/RF in RF

- These expansions are always possible whether or not $n=p$

- If $n=p$, we say "holonomic r^{th} partial ..."

- However, if there are m motion constraints, then only $p=n-m$ of G.S.'s are independent

In this case we can use only p independent G.S.'s

$$\bar{V} = \sum_{r=1}^p \tilde{V}_r u_r + \tilde{V}_t$$

$$\bar{\omega} = \sum_{r=1}^p \tilde{\omega}_r u_r + \tilde{\omega}_t$$

r^{th} non-holonomic partial (ang.) velocities of point/RF in RF
(constrained)

All partial velocities are partial derivatives of velocities with respect to the corresponding G.S.'s

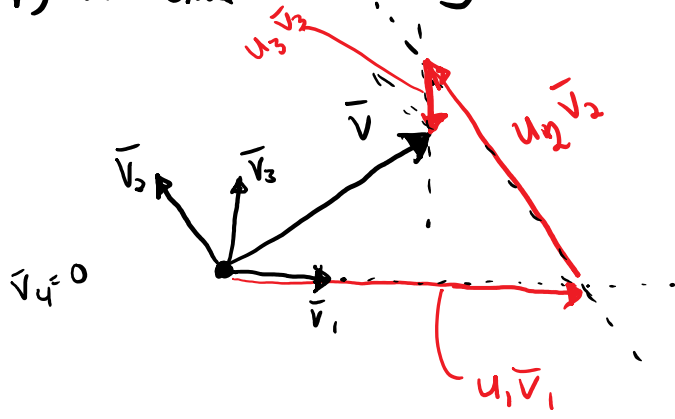
$${}^A \frac{d\bar{p}}{dt} = \sum_{i=1}^n {}^A \frac{\partial \bar{p}}{\partial q_r} \dot{q}_r + \frac{\partial \bar{p}}{\partial t}$$

$$\text{and } {}^A \frac{d\bar{p}}{dt} = \sum_{r=1}^n {}^A \bar{V}_r^P u_r + \bar{V}_t^P$$

$u = 1$

Meaning of partial velocities

1) Kinematic meaning



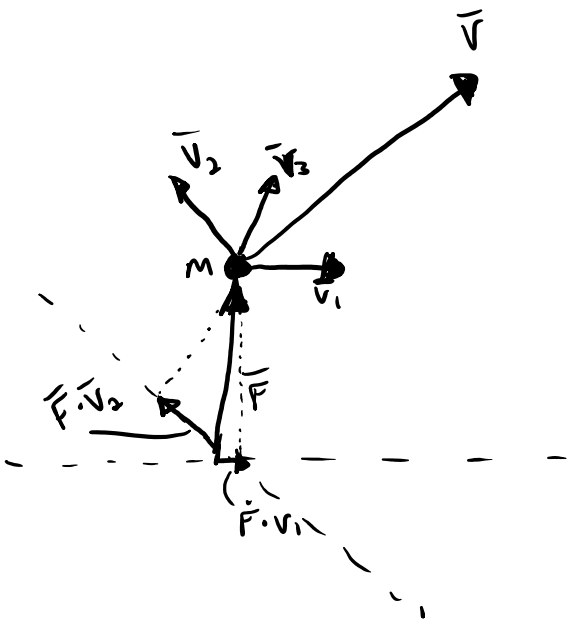
\bar{v} is velocity vector
 suppose 4 G.C.'s, 4 G.S.'s
 calculate 4 \bar{v}_r 's

partial velocities encode the sensitivities of \bar{v} to unit changes in the G.S.'s

2) dynamic meaning

Suppose \bar{v} is the velocity vector of a particle of mass m acted on by force \bar{F} that causes an acceleration $\bar{a} = \frac{d\bar{v}}{dt}$, which occurs with changes in the u 's.

$$\frac{\bar{F} \cdot \bar{v}_r}{m} = \dot{u}_r$$

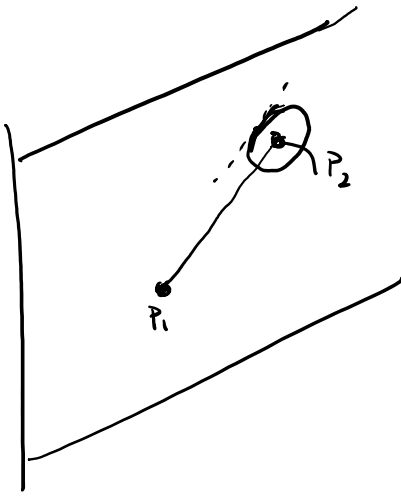


\bar{v}_r is the direction or component of \bar{F} that causes u_r to change, i.e. \dot{u}_r

When a partial velocity $\bar{v}_r \perp \bar{F}$ then \bar{F} doesn't

When a partial velocity $\bar{v}_r \perp \bar{F}$ then \bar{F} doesn't
cause that u_r to change! ($\dot{u}_r = 0$)

Example



Replace P_2 with a wheel
that cannot move \perp to
the $P_1 \leftrightarrow P_2$ line. Gives
rolling without slip motion
constraint.