

$$\bar{V} = 1\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$$

$$\bar{Q} = 1\hat{a}_x\hat{a}_x + 2\hat{a}_y\hat{a}_y + 3\hat{a}_z\hat{a}_z$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_A = 1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_A + 2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_A + 3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_A$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_A = 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_A + 2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}_A + 3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}_A$$

Unit Dyadic let $\hat{a}_1, \hat{a}_2, \hat{a}_3$ be mutually \perp unit vectors

$$\hat{U} = \hat{a}_1\hat{a}_1 + \hat{a}_2\hat{a}_2 + \hat{a}_3\hat{a}_3 \triangleq \text{unit dyadic (identity tensor/matrix)}$$

useful properties:

$$\begin{aligned} \bar{V} \cdot \hat{U} &= \bar{V} \cdot \hat{a}_1\hat{a}_1 + \bar{V} \cdot \hat{a}_2\hat{a}_2 + \bar{V} \cdot \hat{a}_3\hat{a}_3 \\ &= v_1\hat{a}_1 + v_2\hat{a}_2 + v_3\hat{a}_3 = \bar{V} \end{aligned}$$

$$\hat{U} \cdot \bar{V} = \bar{V}$$

Inertia application of dyadics

Return inertia vector:

$$\bar{I}_a = \sum_{i=1}^N m_i \bar{p}_i \times (\hat{n}_a \times \bar{p}_i)$$

Use triple vector product identity:

$$\bar{A} \times (\bar{B} \times \bar{C}) = \bar{B} (\bar{A} \cdot \bar{C}) - \bar{C} (\bar{A} \cdot \bar{B})$$

so

$$\bar{I}_a = \sum_i m_i (\hat{n}_a \cdot \hat{U} \bar{p}_i \cdot \bar{p}_i - \hat{n}_a \cdot \bar{p}_i \bar{p}_i)$$

$$\bar{I}_a = \sum_{i=1}^{\nu} m_i (\hat{n}_a \cdot \hat{U} \bar{p}_i \cdot \bar{p}_i - \hat{n}_a \cdot \bar{p}_i \bar{p}_i)$$

$$\bar{I} \triangleq \sum_{i=1}^{\nu} m_i (\hat{U} \bar{p}_i \cdot \bar{p}_i - \bar{p}_i \bar{p}_i) \quad \text{inertia dyadic}$$

$$\bar{I}_a = \hat{n}_a \cdot \bar{I}$$

\bar{I} is the inertia dyadic of S' relative to O .

\bar{I} does not involve any unit vectors like \bar{I}_a or I_{ab} and \bar{p}_i can be expressed in any coordinate system in any reference frame.

$$I_{ab} = \bar{I}_a \cdot \hat{n}_b = (\hat{n}_a \cdot \bar{I}) \cdot \hat{n}_b = \hat{n}_a \cdot \bar{I} \cdot \hat{n}_b$$

For a general dyadic, pre and post multiplication by an arbitrary vector \bar{v} give different results. However inertia dyadic is symmetric and produces the same result in pre and post multiplication.

See IPYNB for SymPy demo.

There is only one inertia dyadic even though there are a infinite # of inertia matrices, since each matrix depends on a choice of some reference. An inertia matrix is a representation of an inertia dyadic in some coordinate system.

Parallel Axes Theorem

When O is the center of mass of S , then \bar{I} is the "central inertia dyadic". There is a relationship among inertia dyadics about an arbitrary point Q and the central inertia dyadic.

$$\bar{I}^{S/Q} = \bar{I}^{S/S_0} + \bar{I}^{S_0/Q}$$

inertia dyadic of S for Q inertia dyadic of S for mass center inertia dyadic relative to Q of a particle S_0 with mass of S

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Principal Axes

In general the inertia vector is not parallel to \hat{n}_a . But sometimes it is and when it is, then \hat{n}_a and its line L are called a principal direction or principal axis of S for O . The plane perpendicular

principal axis of S for O . The plane perpendicular to \hat{n}_a is called a principal plane. The moment of inertia scalar about \hat{n}_a is the principal moment of inertia. If $\hat{S}_i, i=1, \dots, 3$ are the principal directions then you can write:

$$\bar{I}^{S/O} = I_1^{S/O} \hat{S}_1 \hat{S}_1 + I_2^{S/O} \hat{S}_2 \hat{S}_2 + I_3^{S/O} \hat{S}_3 \hat{S}_3$$

3 principal moments of inertia and 3 principal axes of S for point O .

Equivalent to

$$\bar{I}^{S/O} = \begin{bmatrix} I_1^{S/O} & 0 & 0 \\ 0 & I_2^{S/O} & 0 \\ 0 & 0 & I_3^{S/O} \end{bmatrix}$$

products of inertia are zero

The principal moments of inertia are the eigenvalues of an inertia matrix and the principal axes are the eigenvectors of an inertia matrix.